

# Lecture 10: Sampling, sweeps, & uncertainty

---

Complex Systems 530

3/10/20

# Welcome back!

- Networks lab check in
- Final project details
- Plan for the next few weeks

# Final Project: Presentation

- **Dates**
- **Length:** tentatively, we will plan for (potentially to be changed depending on scheduling needs):
  - 1 person: 10 minutes + 2 for questions
  - 2 people: 12 minutes + 2 for questions
- Everyone in the group must present
- Ask at least one question of another group (or more!)

# Final Project: Paper

- **Due date: Final exam day, 4/30/2020**, via Canvas (grades are due 72 h later, so this is a hard deadline!)
- **Write-up:** roughly 8-12 pages, but shorter or longer is fine so long as you fully cover the motivation, problem, methods/model, results, etc.
- **Model code:** I should be able to both review the code and run it myself if in NetLogo or Python.
  - Your code must be **documented, clear, and readable.**
  - Be sure to also document the version of python and any packages you used.

# Paper components

- **Introduction.** problem overview and literature review—what gap or question are you addressing? What has been done before?
- **Methods: model description.** Describe how your model works in terms of its: agents, interactions, environment, model schedule/timing
  - You can use the PARTE and/or ODD frameworks as a guide
  - Flow charts & visuals are good!

# How to write up an ABM

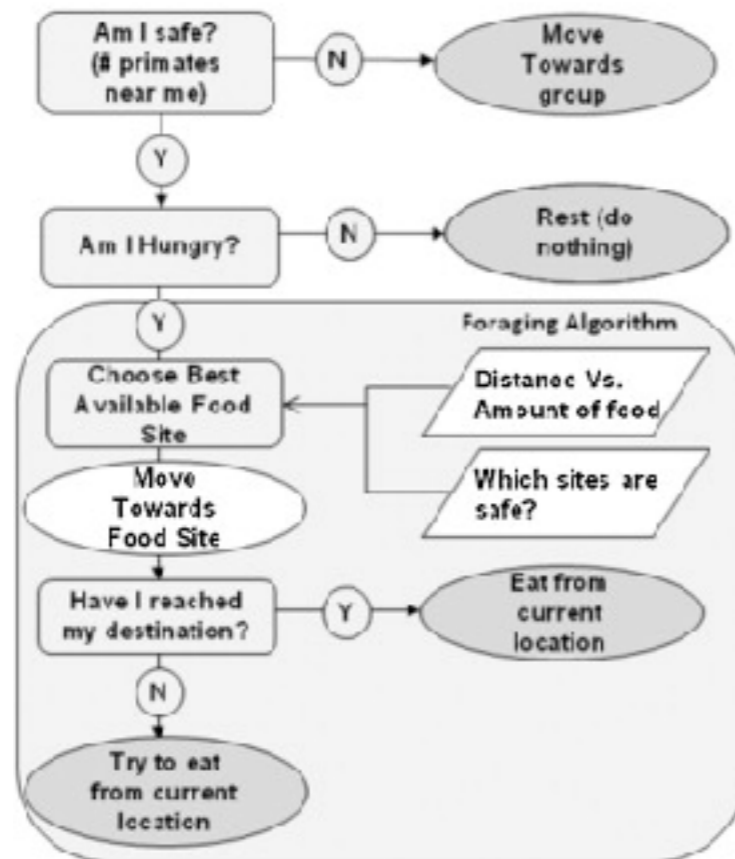
- PARTE - Properties, Actions, Rules, Time, Environment
  - Agents are defined by their properties, actions, and rules
- ODD Protocol - Overview, Design concepts, and Details
  - Describes the model purpose, structure, and design concepts around which the model was built (e.g. emergence, adaptation, etc.)
- You will get more practice on the next lab!

# Flow Chart Example

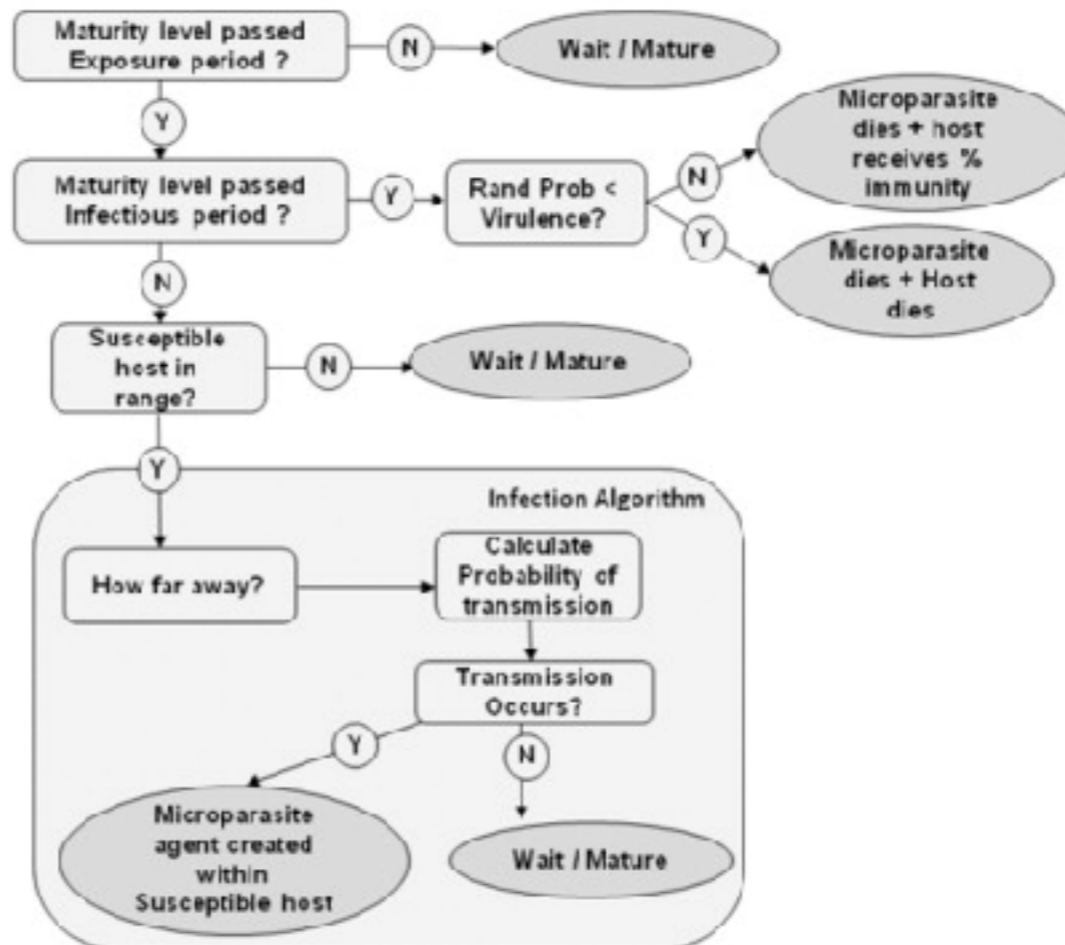
Simulation model



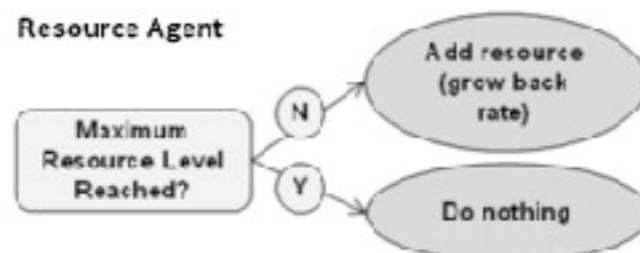
Primate Agent



Microparasite Agent



Resource Agent



Bonnell et al., 2010. An agent-based model of red colobus resources and disease dynamics implicates key resource sites as hot spots of disease transmission. <https://www.vetmed.wisc.edu/goldberglab/pdf/P071.pdf>

# Paper components

- **Methods: model analysis.** Describe the parameter settings you swept through, the analyses you ran. If your model is stochastic, you may need to run multiple trials at the same parameter settings.
- **Results.** Provide qualitative and quantitative summaries of how your model behaves. Provide graphs and plots of model outcomes at different settings as needed.



# Paper components

- **Discussion.** Return to the question or problem in your intro—what do your results say about this problem? Put this in a broader context. Describe the strengths, limitations, and potential future directions of your work.
- If you think there are still some bugs driving your model's behavior, this is the place to discuss this
- Also a good place to talk about how you might verify, validate, or extend the model in the future

# A note about verification as you build your model

- Your model will have bugs, be wrong, do things you didn't intend/think through, etc.
- Unit testing and thinking and building with small examples and modules is key
- Figure out small test cases where almost the entire model is either absent or set to a constant and you know what the one piece you're working with should do. Then move on to the next piece/submodel, etc.

# Plan for the next few weeks

- Sensitivity analysis and exploring the parameter space
- Visualizing and interpreting results
- Different levels of agent cognition - game theory & decision theory
- Model calibration & estimating parameters from data
- Model comparison & inference robustness
- More complex environments (e.g. GIS)

# ABM complexity & dimensionality

# ABM complexity & issues of dimensionality

- So far, our ABMs have obeyed fairly simple rules—reasonably easy to describe with a bulleted list (even if this can leave a lot of ambiguity still)
- This already leads to a high dimensional input/parameter space
- But many ABMs used in practice include more detailed mechanisms, more complicated processes to capture realistic details or more intricate questions

# Parameter/input space

- Voting model
  - Grid size (width, height)
  - Initial fraction/probability of yes/no votes
  - Neighborhood size

# SIR model on a network

- Network generation parameters (e.g. total number of nodes and probability of an edge for an Erdős-Renyi graph)
- Probability of transmission
- Probability of recovery
- Initial conditions (numbers of S, I, and R individuals)

# Power grid network model

- Network parameters: number of nodes, number of connections for each new node (scale free graph)
- Node capacity distribution parameters (in this case the mean and SD for a normal distribution)
- Initial electrical load parameters (in this case the bounds for a uniform distribution)
- Initial node states (e.g. fraction of failed nodes at start time)



# Ants Model

- Food sources: number of sources, sizes, locations
- Nest location, nest scent gradient parameters
- Population size
- Chemical (pheromone) parameters: how much, diffusion rate, evaporation rate
- Ant movement parameters (angles to wiggle), chemical detection upper & lower bounds, chemical and nest scent angle bounds, how much chemical to drop, etc.

# Example: Walking & Health

## A Spatial Agent-Based Model for the Simulation of Adults' Daily Walking Within a City

Yong Yang, PhD, Ana V. Diez Roux, PhD, MD, Amy H. Auchincloss, PhD,  
Daniel A. Rodriguez, PhD, Daniel G. Brown, PhD

- Models walking behaviors in an Ann Arbor-sized city as a function of health, SES, neighborhood safety, commutin patterns, and other features
- Time is discrete steps, with 1 step = 1 day

# Environment

- Environment: 800 x 800 grid
- Each cell = 10 m x 10 m  
(64 km city ~ Ann Arbor)
- Neighborhoods - 40 x 40 cells
  - Neighborhood properties: safety, aesthetics, other environmental elements

# Environment properties

- Elements in the environment (based on Ann Arbor census)
  - 200 groceries
  - 800 non-food shops
  - 1500 social places
  - 12000 workplaces
  - 60000 households

# Agents

- 108,000 individuals
  - 48,000 couples, and 12,000 single people
  - Range of properties related to walking, SES, health, etc.
- E.g. walking ability is determined by:

$$A_b = U^4(0, 1) \times \left( \frac{\text{Min}(|137 - \text{Age}|, 100)}{100} \right)$$

**Table 1.** Properties of individuals and selected model parameters

<b>Individual-level properties</b>	<b>Values and meaning</b>	
<b>Gender</b>	Assigned as male or female with equal probability	
<b>Age</b>	Random integer from the uniform distribution ranging between 18 and 87 years; the difference between a couple is no more than 3 years	
<b>SES</b>	Integer value ranging between 1 and 5, with higher values indicating higher SES. Members of a family are assumed to have the same SES. In select scenarios, SES is assumed to be positively correlated with increasing distance from the city's center	
<b>Family size</b>	Family size can be 1 (with probability of 20%) or 2 (with probability of 80%)	
<b>Friends</b>	Each individual has 3 to 5 friends who can influence her/his walking attitude, randomly selected from the people with the same or similar SES value (difference no more than 2)	
<b>Dog ownership</b>	Each individual has a 20% probability of having a dog (dog owners have a higher probability of walking within the neighborhood)	
<b>Household</b>	Each person is randomly assigned to a household (except in scenarios involving residential segregation by SES)	
<b>Work and workplace</b>	If aged $\leq 69$ years, the probability of working is 95%; people aged $>69$ years are assumed to not work. Working people are randomly assigned a workplace in the city. This can be a grocery store, a non-food shop, a social place, or another workplace in the city	
<b>Walking ability (<math>A_b</math>)</b>	Value ranges from 0 to 1, the higher the value, the longer distance an individual can walk	
<b>Attitude toward walking (<math>A_t</math>)</b>	Value ranges from 0 to 1, the higher the value, the higher probability an individual will walk	
<b>CALIBRATED MODEL PARAMETERS</b>		
<b>Activity</b>	<b>Daily probability of performing the activity</b>	<b>Maximum walking distance for the activity (miles)</b>
Work <sup>a</sup>	1 for individuals with a job; 0 for others	1.125
Food shopping	0.4	2
Other shopping	0.25	1.5
Visiting a social place	0.2	2.5
Leisure within neighborhood	0.33	5.5

<sup>a</sup>Only among employed

# Actions & Rules

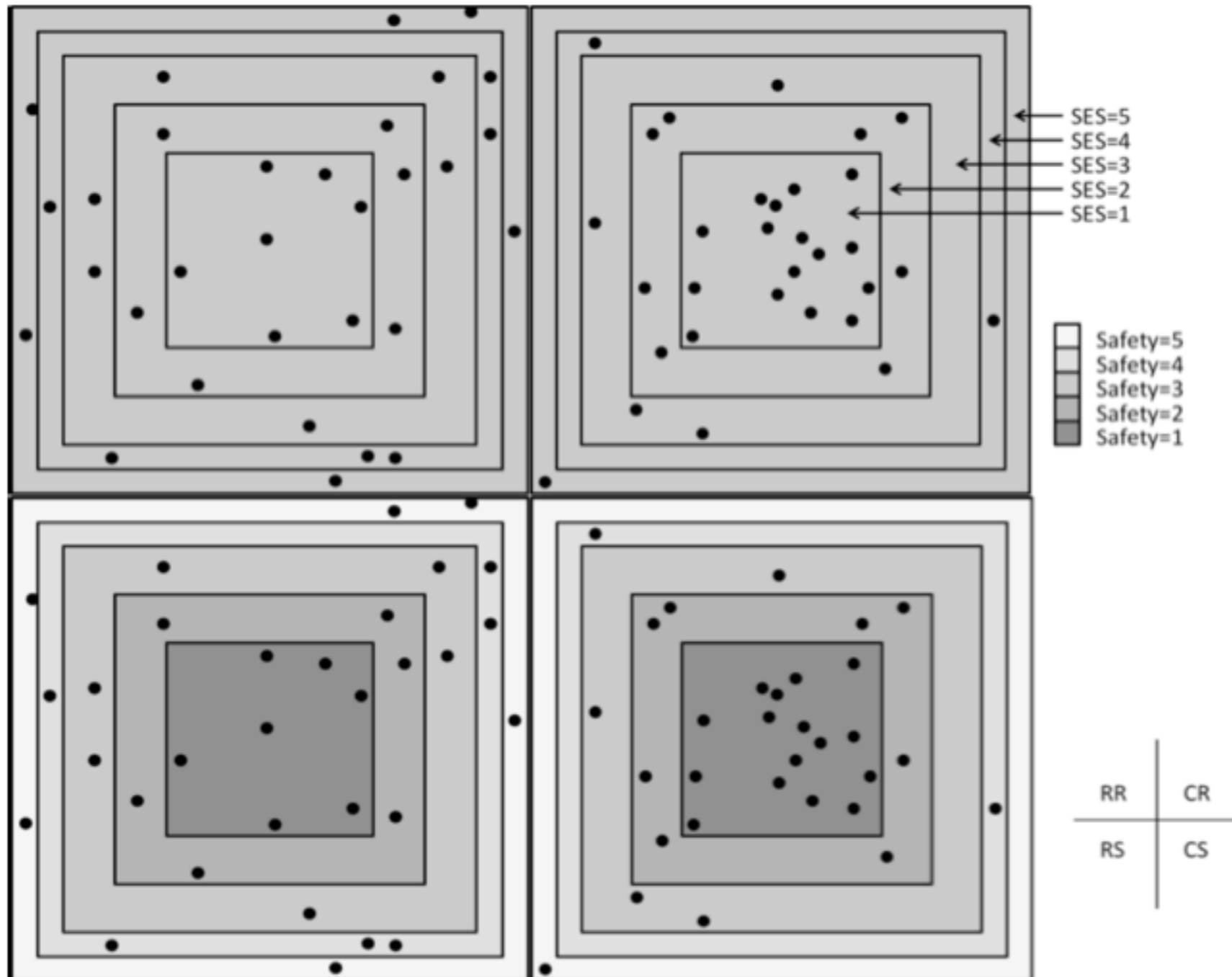
- Individuals can walk for three purposes: (1) to work; (2) for basic needs (such as food shopping, other shopping, and visits to social places); and (3) for leisure.
- Walk with a certain probability each day (e.g., non-food shopping takes place every 4 days on average, resulting in a daily probability of 0.25).
- Maximum walking distance for each type of activity. Person-specific maximum distances are calculated as the product of maximum distances and the person-specific ability ( $A_b$ ).

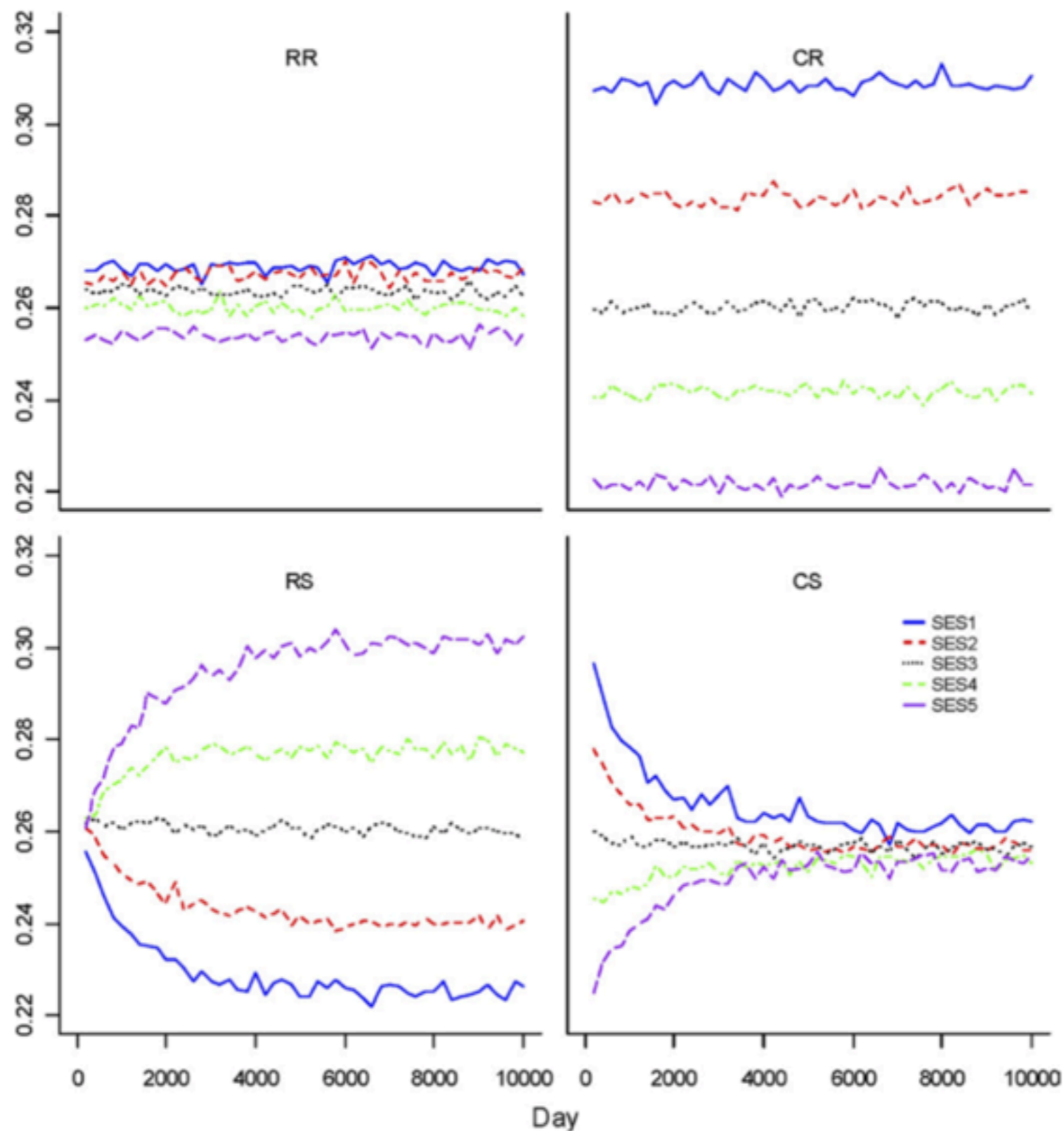
# Actions & Rules

- Example: Work. If the distance between the person's household and workplace is less than the person-specific maximum walking distance for work, then the decision to walk is a random draw with probability equal to attitude  $A_t$ .
- Attitudes update each time step with feedback from household members, safety, aesthetics of the route, etc.



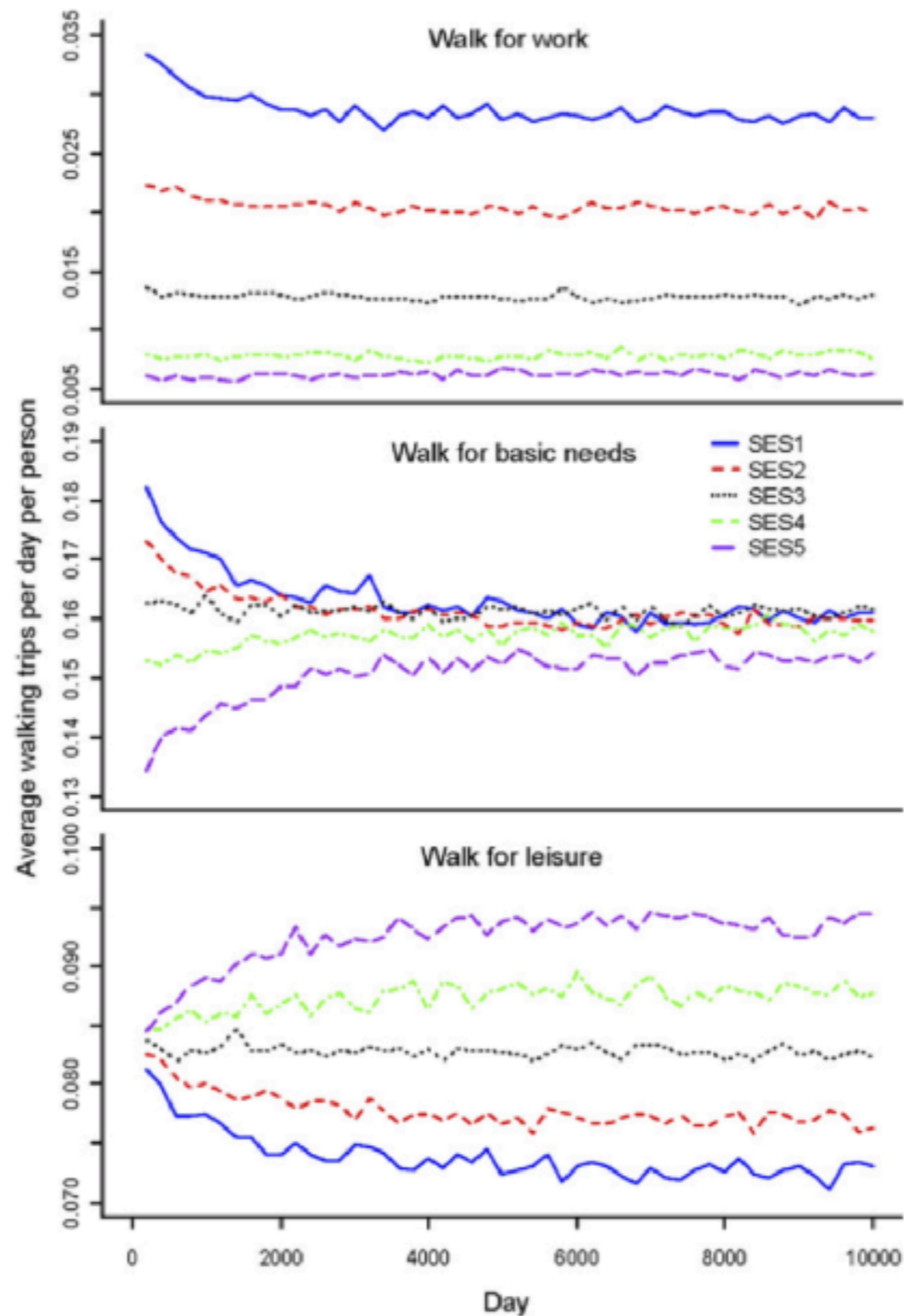
# Example: Walking & Health





**Figure 1.** Average walking trips per day per person for different SES groups over time for four scenarios

CR, more non-household locations in the core and random safety level; CS, more non-household locations and lower safety level in the core; RR, randomly distributed non-household locations and random safety level; RS, randomly distributed non-household location and lower safety level in the core



**Figure 2.** Average walking trips per day per person for different purposes and for different SES groups over time in the CS scenario  
 CS, more non-household locations and lower safety level in the core

# Complexity & high-dimensionality with ABMs

- Many different inputs and possible outputs
- Too complicated to explore by hand
- Need ways to:
  - Visualize and interpret the variation in behaviors
  - Explore which parameters/inputs have the most effect on model outputs of interest

# Complexity & high-dimensionality with ABMs

- Also underscores the importance of being targeted and thoughtful in your question!
- Freedom of design means you can easily end up with way too many variables to analyze effectively
- Decide what scenarios to run (e.g. four city setups), what questions to answer, and then keep it as simple as possible

# High dimensionality of ABM

- Benefit: ability to explore relationships between large spaces of system inputs & system outputs
- Also allows for exploration of stochastic processes and evolution of systems across time and space
- However, very high dimensionality of data to deal with in analysis

# High dimensionality of ABM

Level	Inputs/Parameters	Outputs
Agent	Initial distributions of agent types, attributes, preference structure, direction, number, density	Final distribution of resources, density, proportion in a given state, position
Interaction	Type of strategy, connection topology, adaptation	Strategy prevalence, network structure
Environment	Resources, type, layout	Resources, percent of environment in a given state

# Sensitivity, uncertainty, & sampling



# Goals

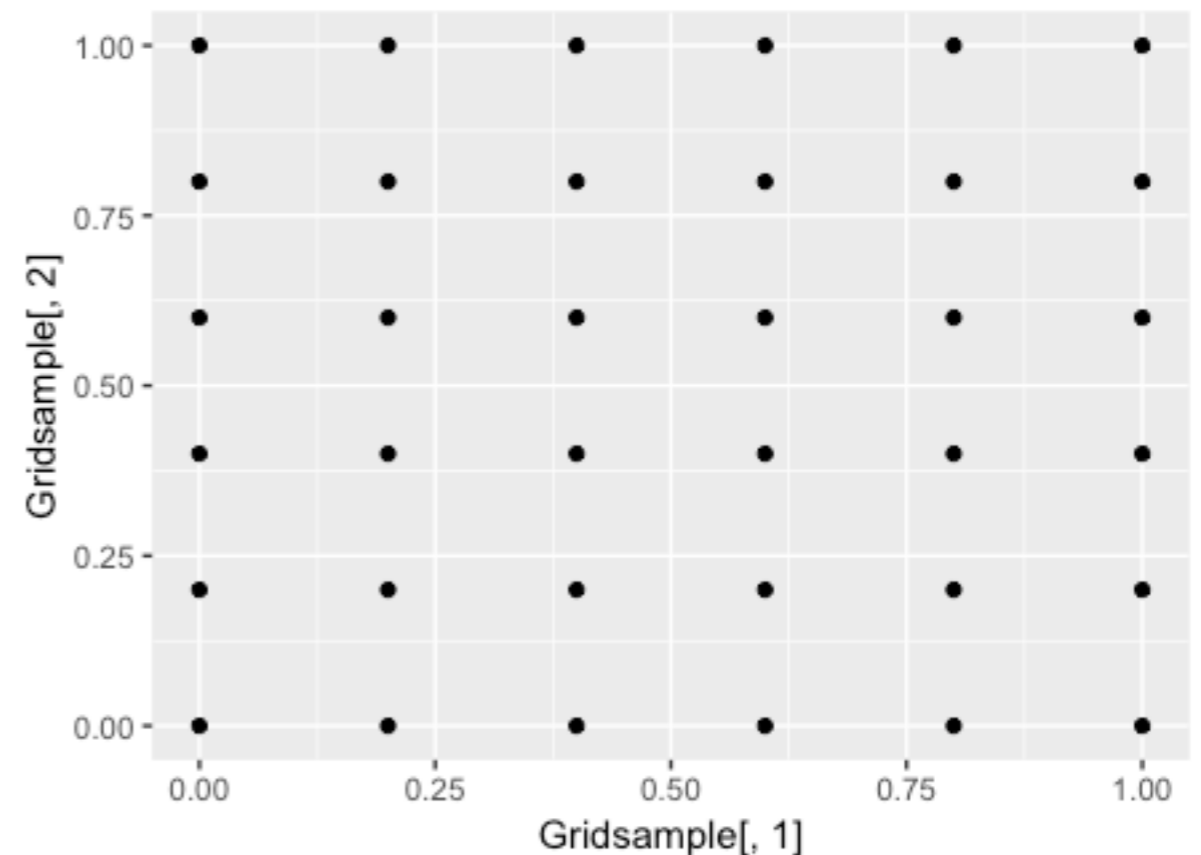
- Understand the variation in model outputs & behaviors (often termed **uncertainty quantification**)
- Understand which inputs/parameters drive that variation (**sensitivity analysis**)

# Exploring the space of model behaviors

- How to explore model behaviors?
- So far, we have focused on adjusting specific variables and parameters, and seeing how the model behavior changed by running the model a few times and visually assessing
- We need a more comprehensive, systematic approach

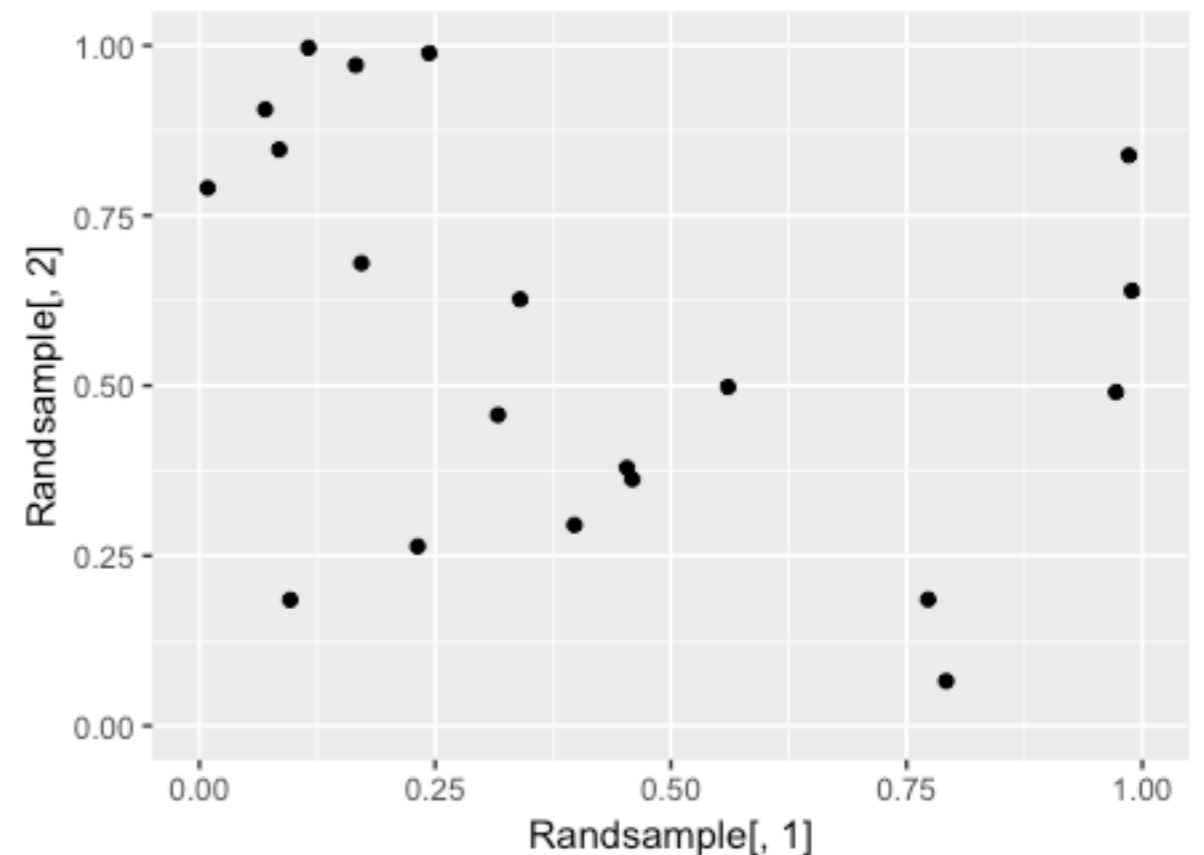
# Sampling parameter space

- **Grid sampling**
- Typically choose uniform distribution of points
- Good coverage of space
- Computationally expensive! Becomes infeasible as dimension increases



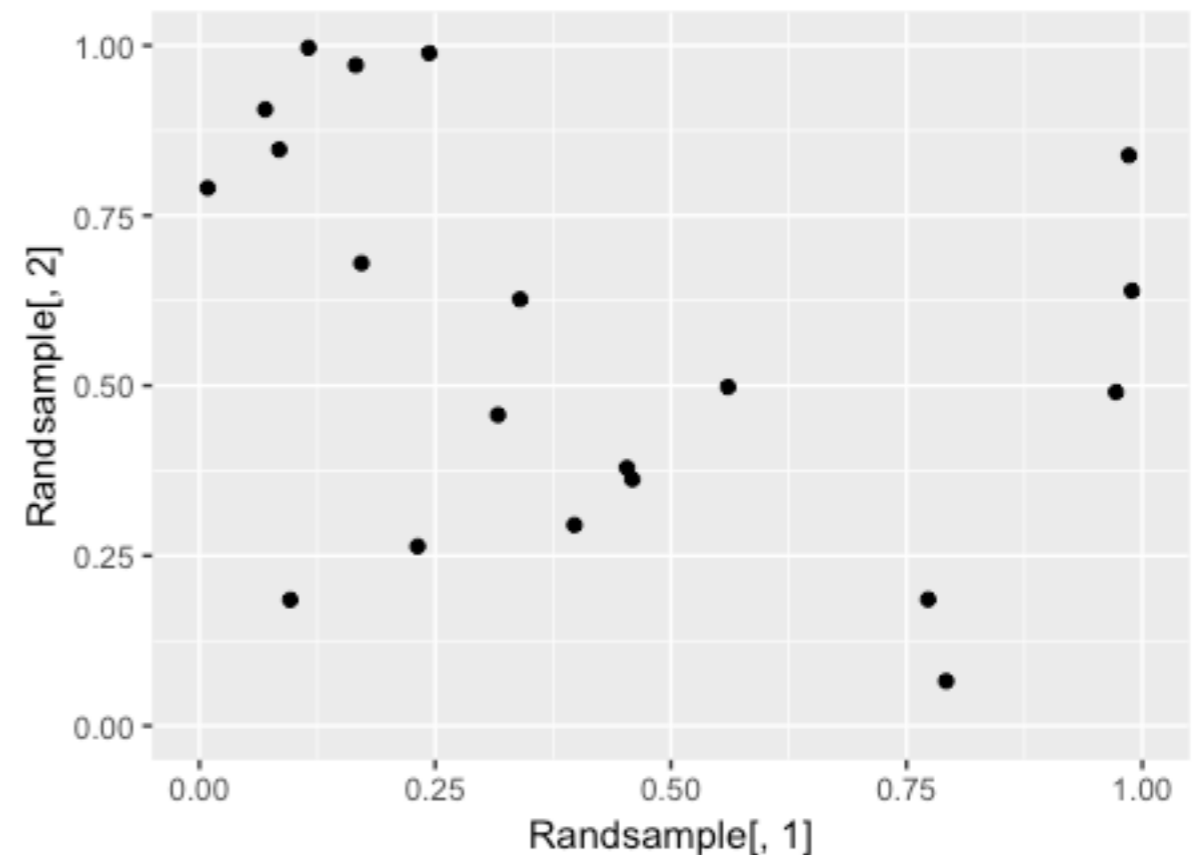
# Sampling parameter space

- **Random sampling**  
(Monte Carlo)
- Often done with uniform distribution, but can choose any distribution
- However, may leave big blank spots, require many samples to fully explore the space



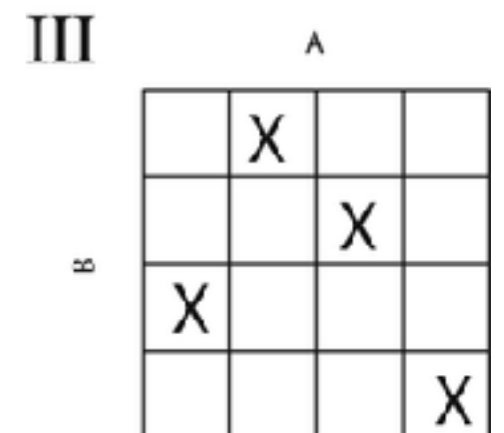
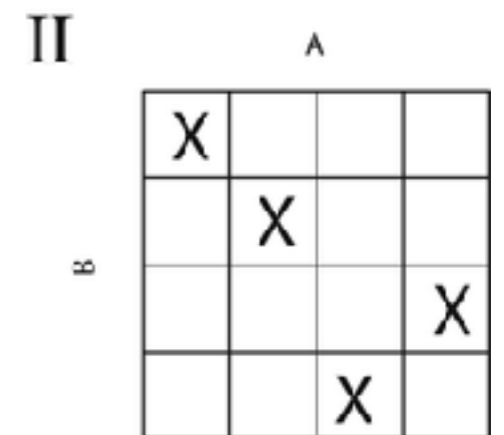
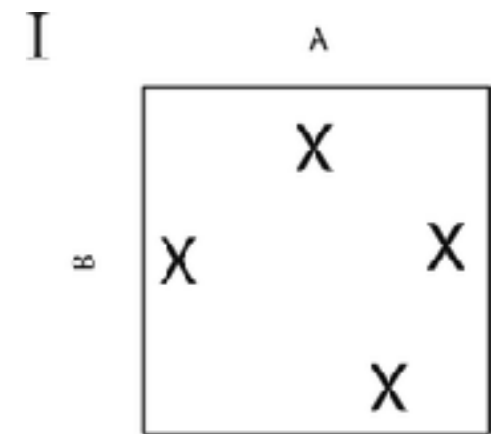
# Sampling parameter space

- More efficient ways to explore the space?
- Latin hypercube sampling (& variants, orthogonal, etc.)
- Sobol sampling (& other low-discrepancy sequences)



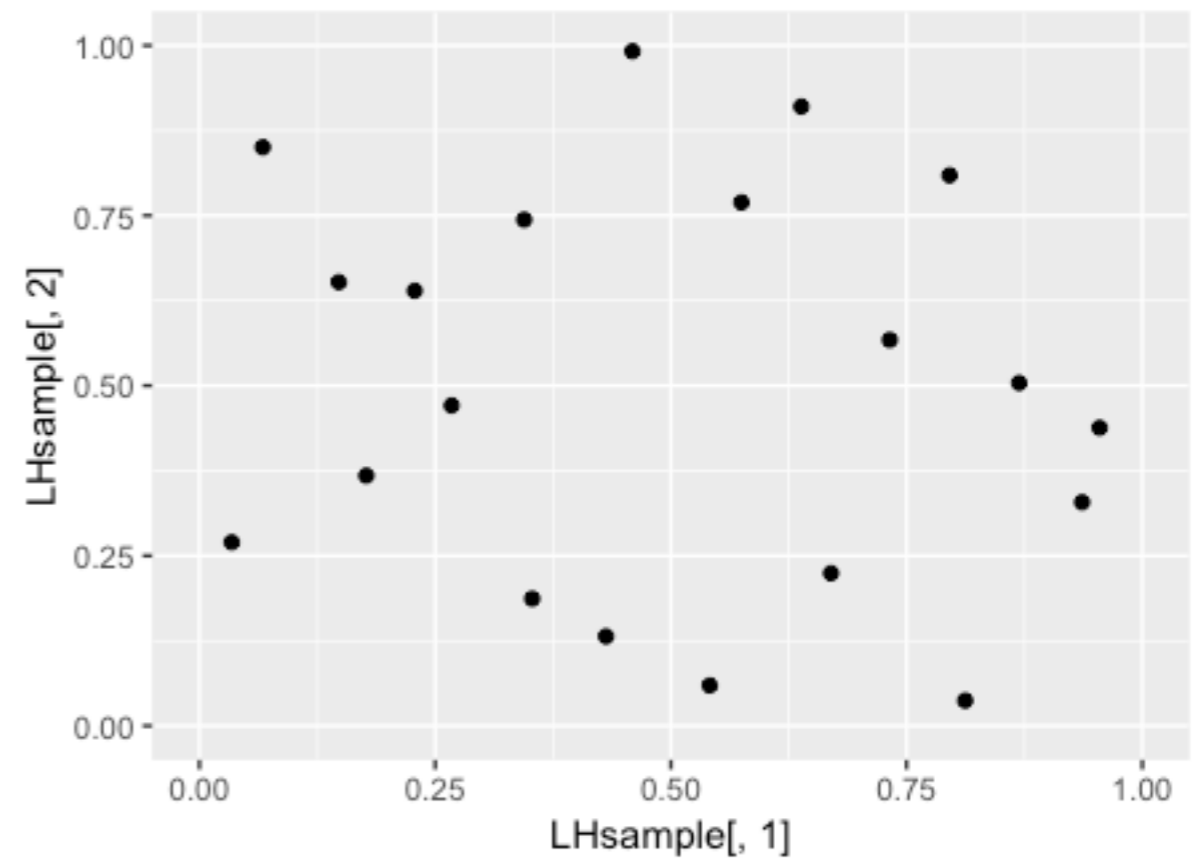
# Latin hypercube sampling

- Kind of like sudoku
- Divide space into a grid of rows & columns
- Choose one square in each row and each column
- Choose a random point within that square



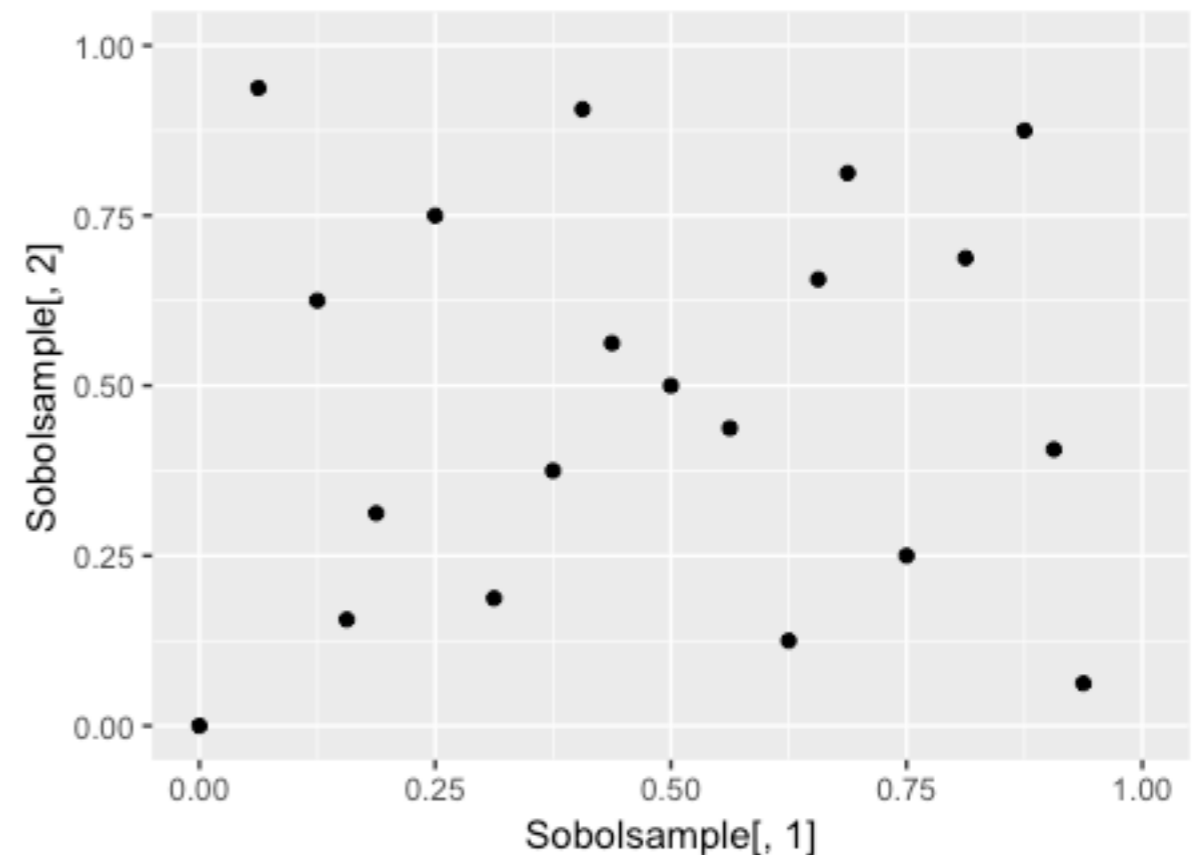
# Latin hypercube sampling

- Still an element of randomness
- Ensures better coverage of the space/faster convergence to the sampled distribution



# Sobol sampling

- Low-discrepancy sequence (see also Halton, Faure)
- Generates a sequence that samples the space evenly but requires few points
- Convergence can be better than LHS





# How many samples to take?

- Tough to say! Balance computational intensiveness with good coverage (often  $\gg 100$ , e.g. in the 1K to 10K range depending on number of parameters)
- May need to run more than one sample for a given point due to stochasticity (since different runs may give different behaviors)
- Some methods have rules of thumb, e.g. for LHS,  $N_S > (4/3) \times N_P$  has been proposed, but you will often want much more than this bound

# Choosing outputs

- Given that we'll be running large numbers of samples, evaluating the model behavior visually will be difficult!
- Choose your model outputs based on your question of interest—be thoughtful about this!
- Can often be hard to pick metrics that accurately capture the behaviors you're interested in examining

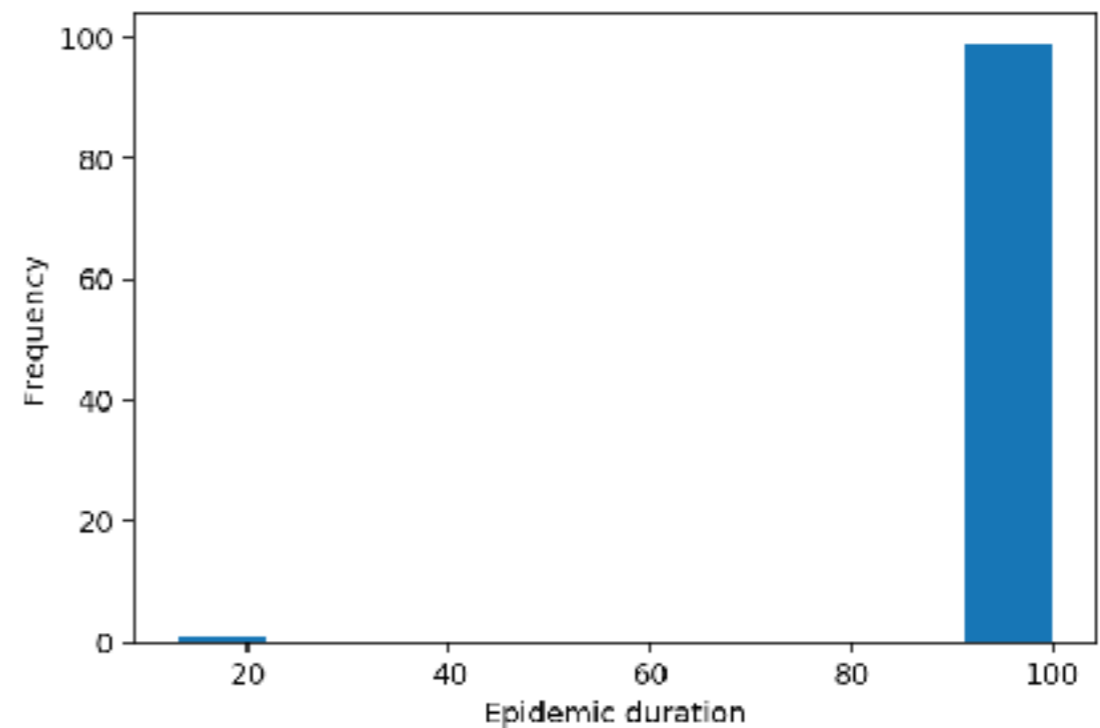
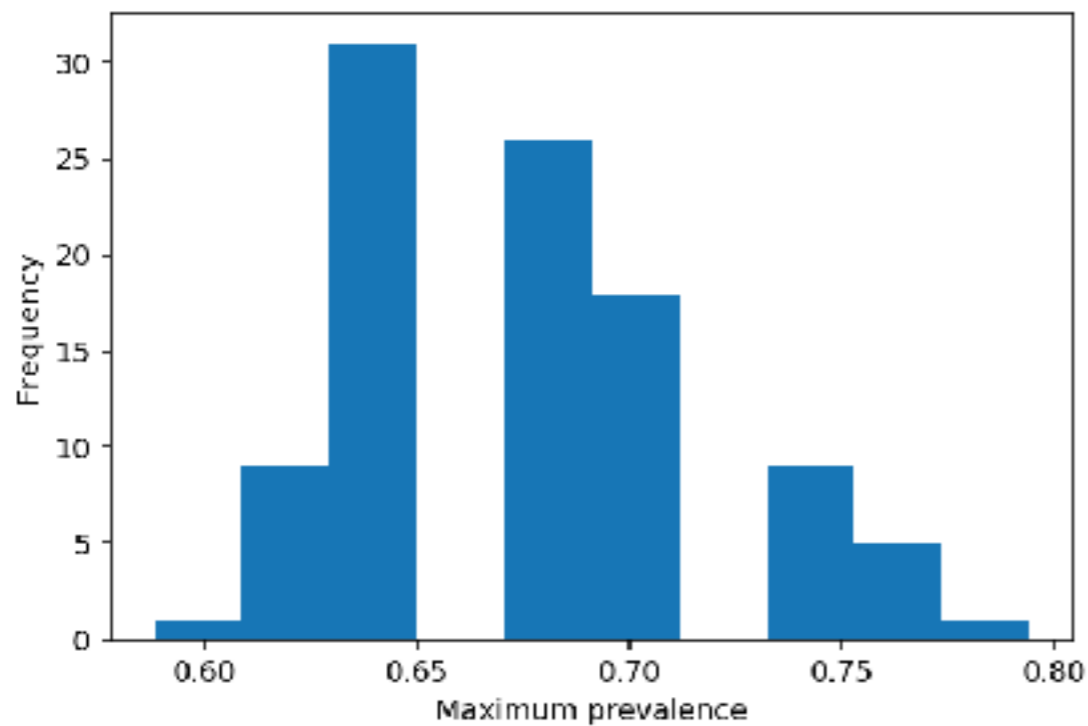
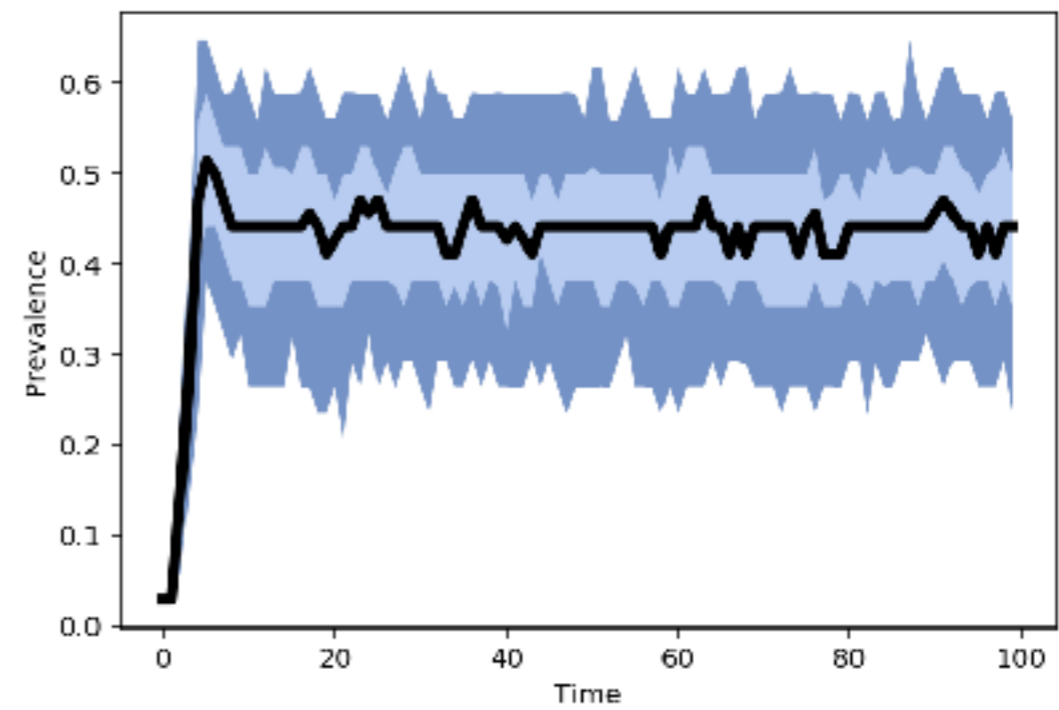
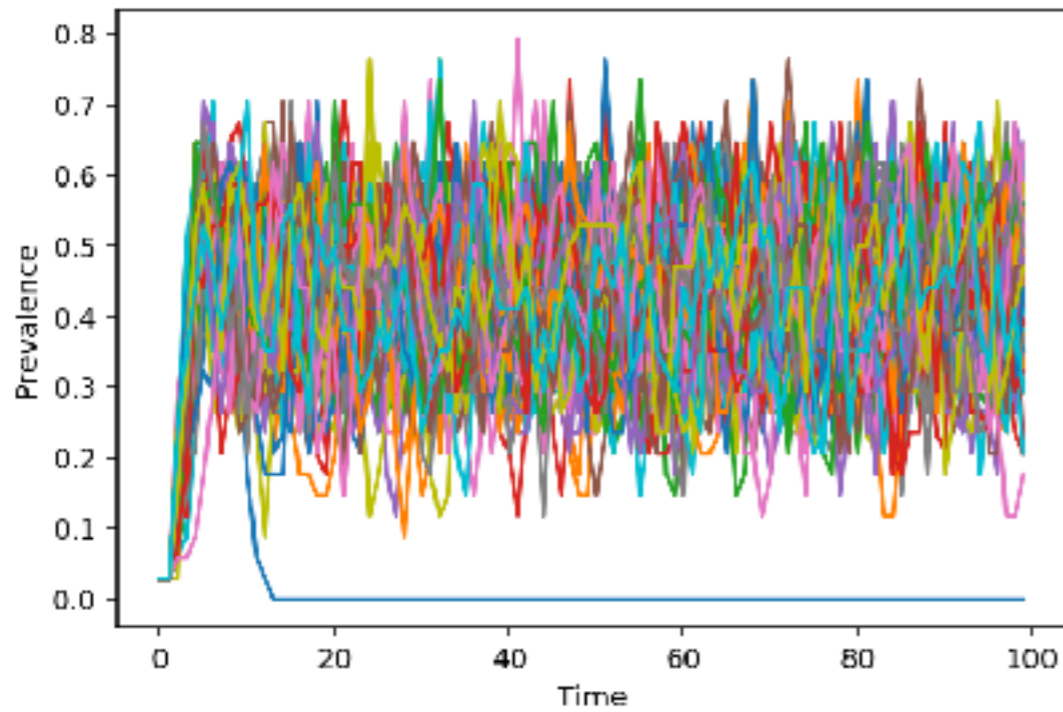
# Visualizing & interpreting results

- Histograms and scatterplots—simple but useful!
- Boxplots, violin plots, ridgeline plots, heatmaps
- Basic descriptive stats (means, medians, SD, quantiles)
- We will explore other/fancier options as we go

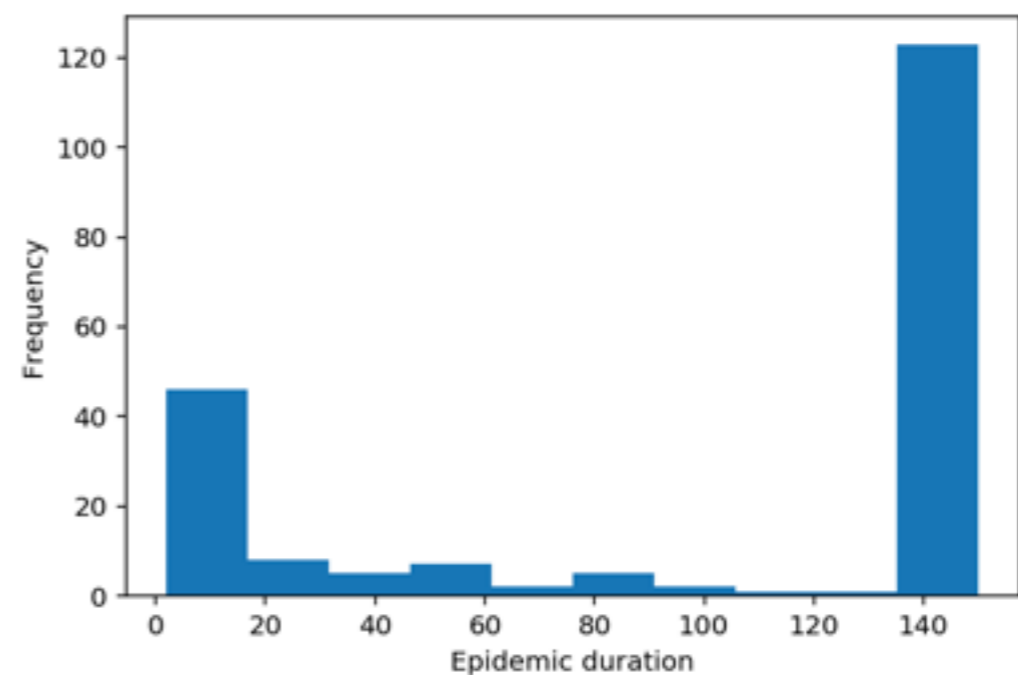
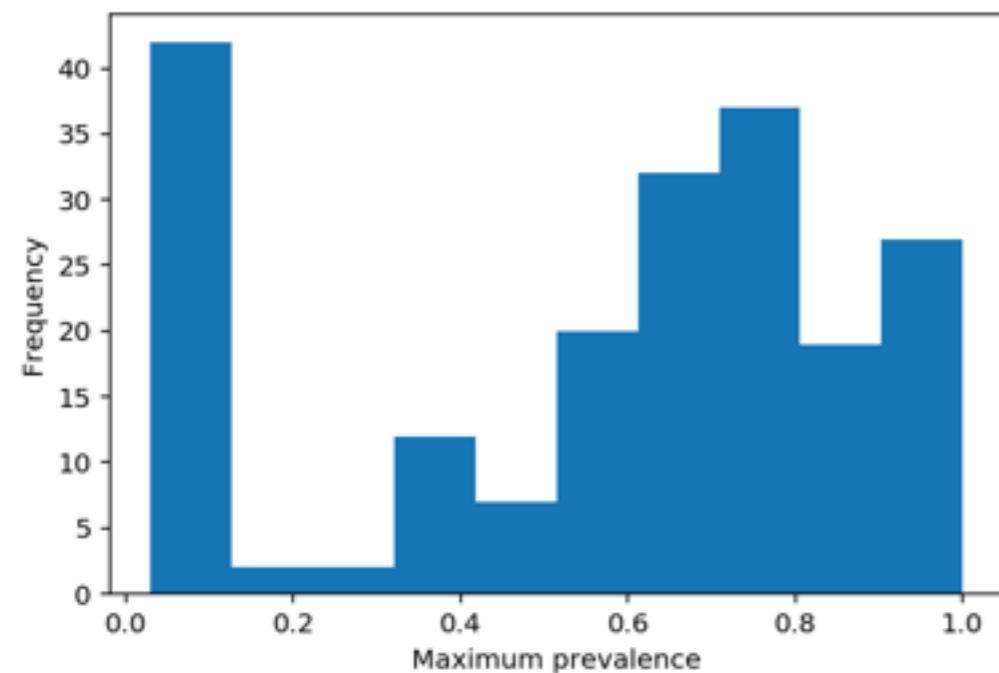
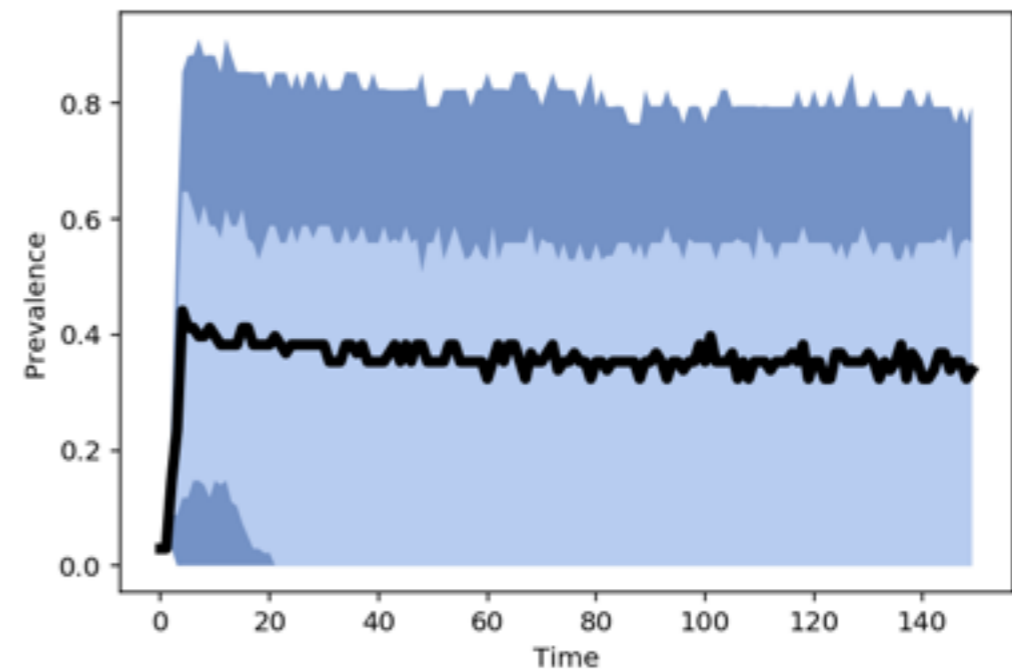
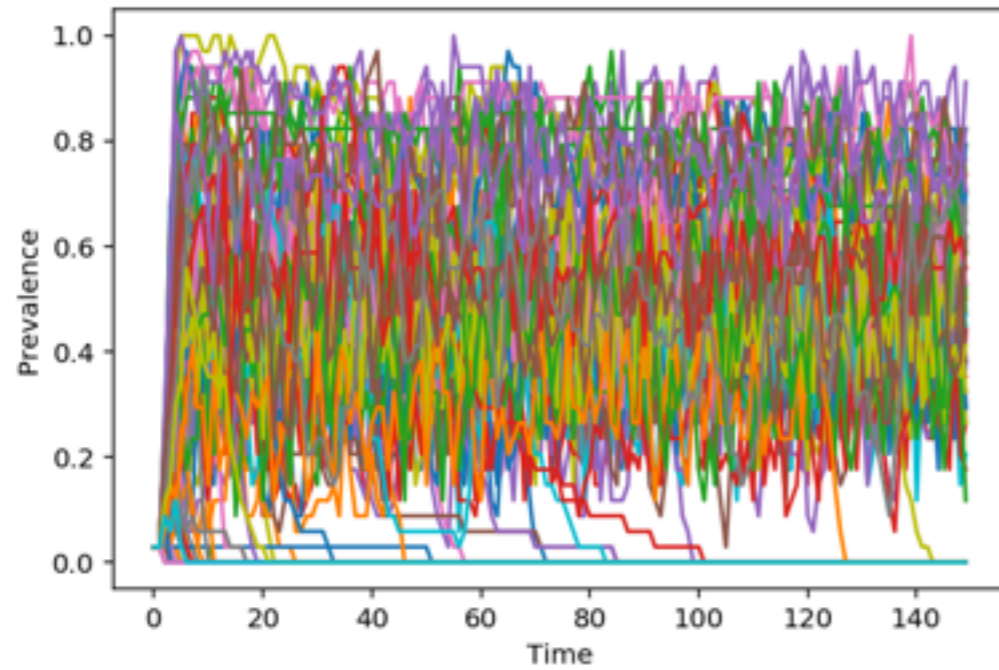
# Example

- SIR on the karate club network, with edge regrowth
- **Parameters to sample:** infection probability, recovery probability, tie-breaking probability, tie-regrowth probability
- **Outputs:** maximum prevalence (fraction infected), duration of the epidemic (within the 100 timesteps simulated)
- Pause to work with code

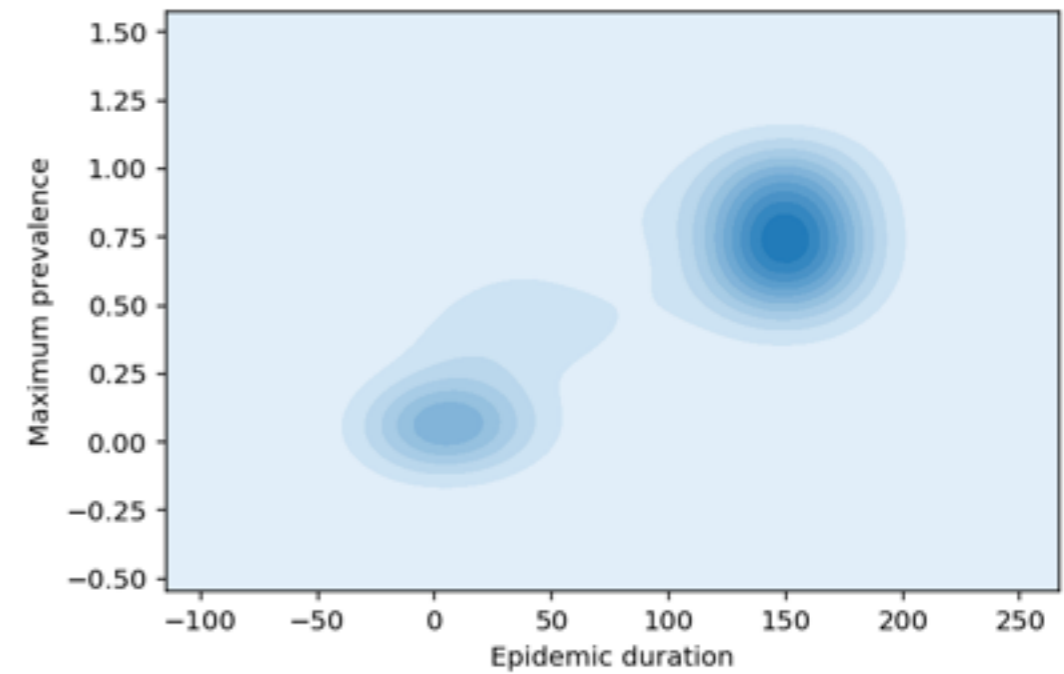
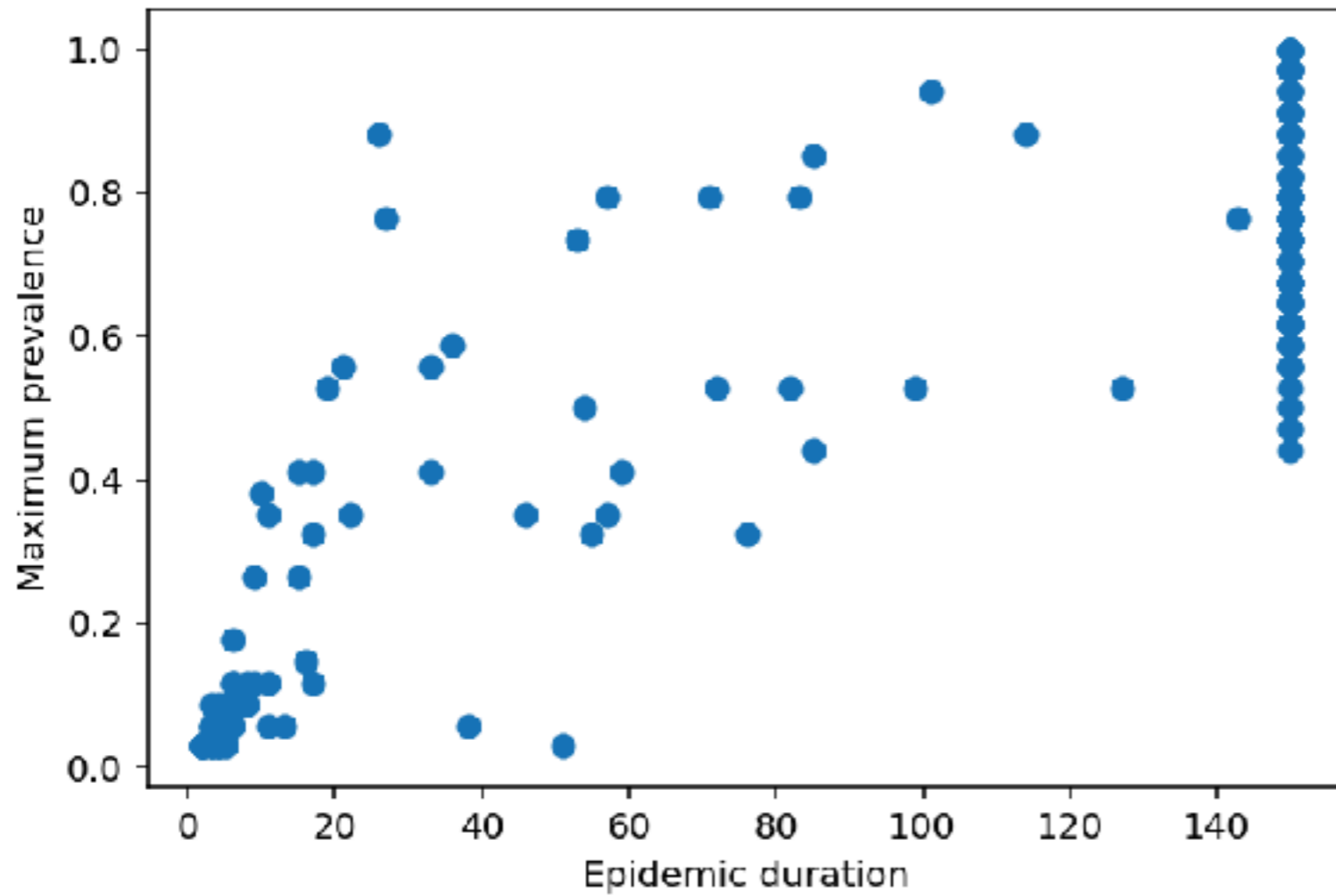
# Stochastic variation



# Sampling parameter space



# Sampling parameter space



# For next time...

- Readings
  - PARTE framework
  - ODD protocol
  - Example ABMs also posted on the website