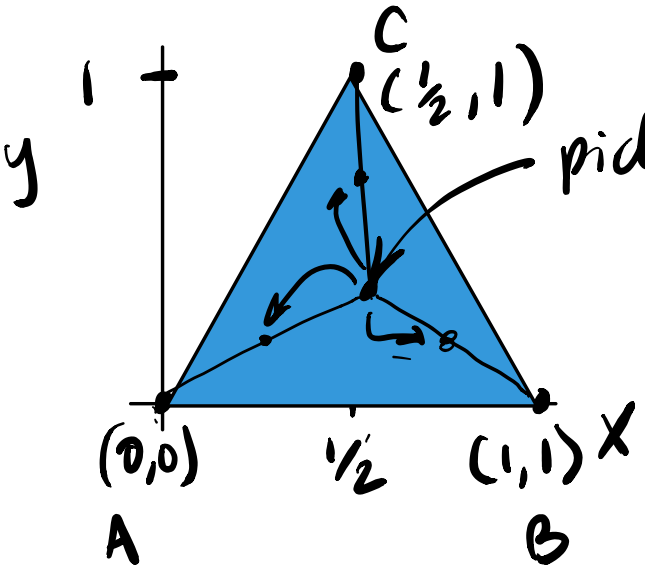
2/11/21

The Chaos Game!



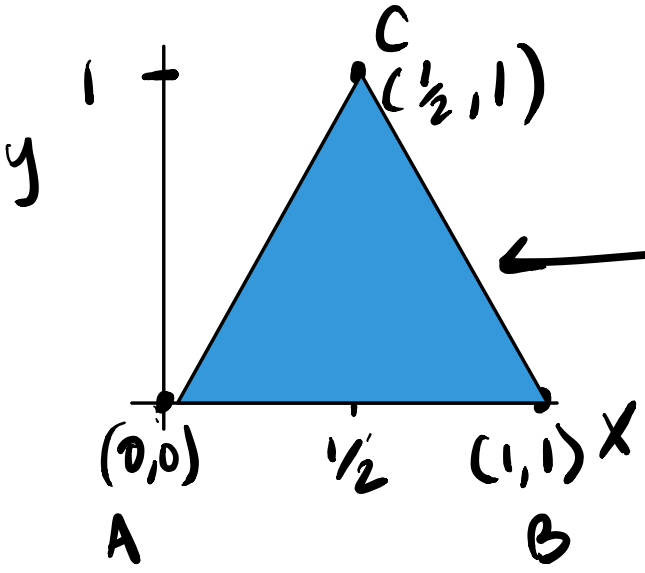
$$f_A(x,y) = \left(\frac{x}{2}, \frac{y}{2}\right) \quad \text{if vertex A}$$

$$f_B(x,y) = \left(\frac{x}{2} + \frac{1}{2}, \frac{y}{2}\right)$$

$$f_C(x,y) = \left(\frac{x}{2} + \frac{1}{4}, \frac{y}{2} + \frac{1}{2}\right)$$

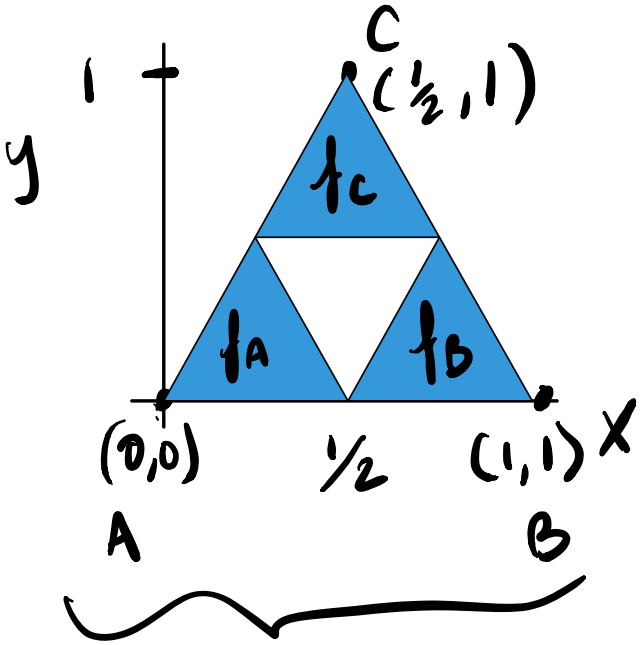
take us halfway to ½

$\frac{x+\frac{1}{2}}{2} = \frac{x}{2} + \frac{1}{4}$



We will:

- Map where every point in here could go
- for all three functions f_A, f_B, f_C



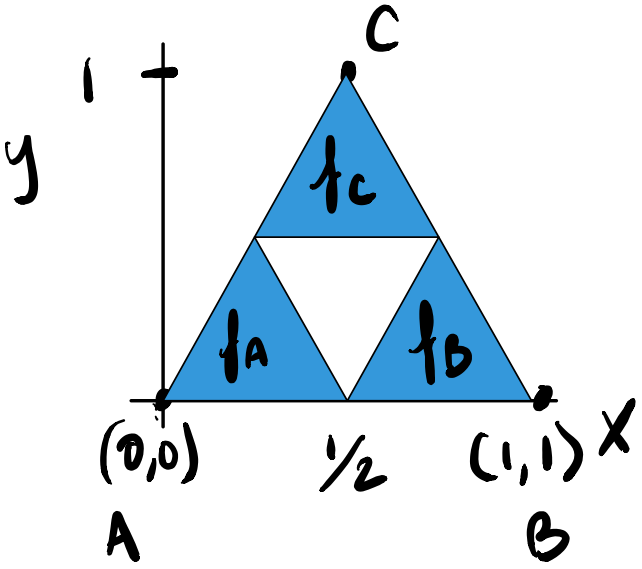
$f_A(A) = f_A(0,0) = (0,0)$

$f_A(B) = f_A(1,0) = (\frac{1}{2}, 0)$

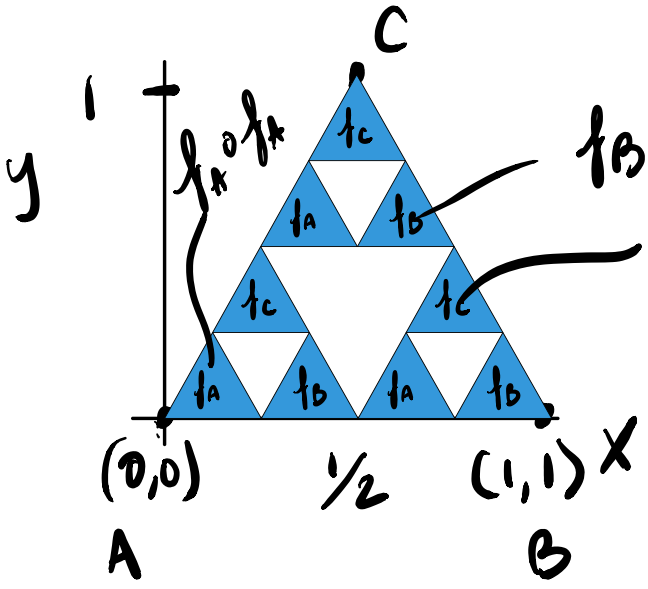
$f_A(C) = f_A(\frac{1}{2}, 1) = (\frac{1}{4}, \frac{1}{2})$

All the places that our original blue triangle could go after 1 step.

What happens after 2 steps?



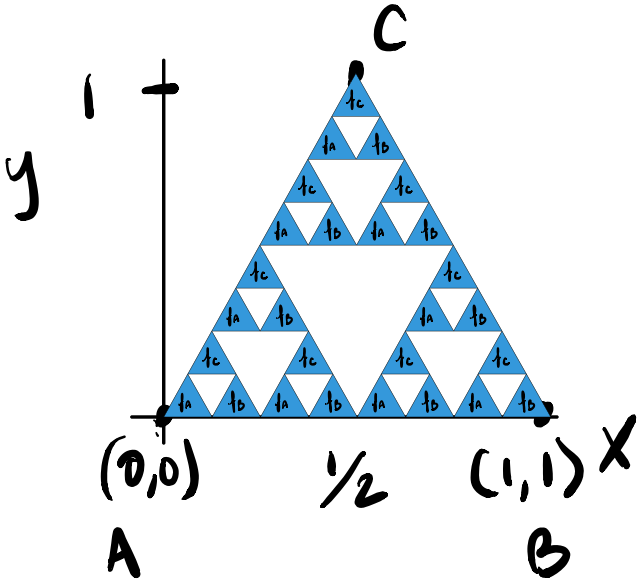
to account for f_A then f_C etc. We apply each function to the whole image.



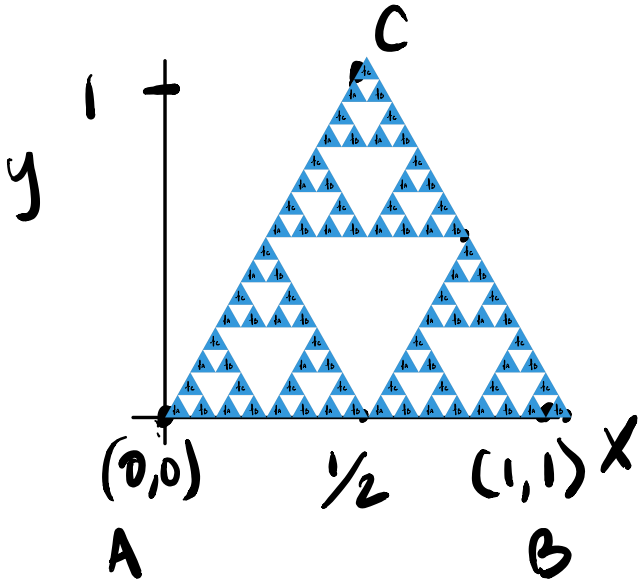
f_B then f_C

f_C then f_B

this picture shows all the places we could be after 2 steps

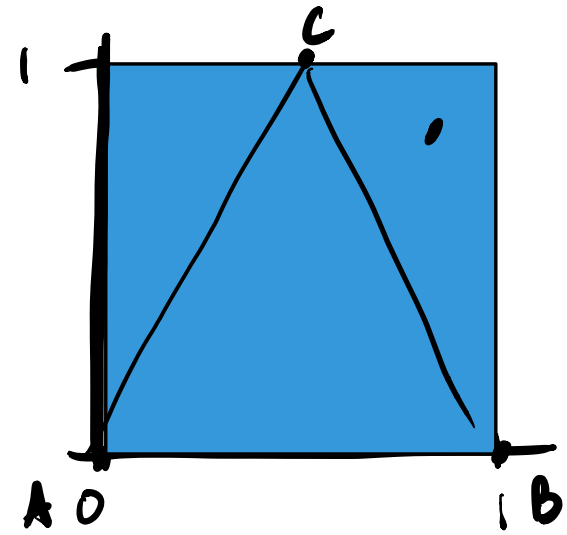


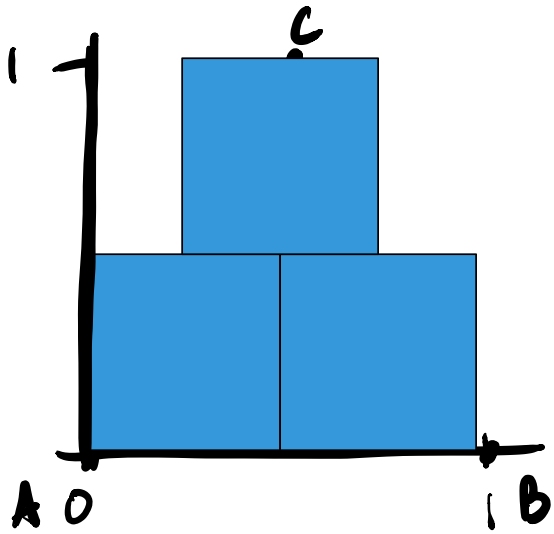
3 steps



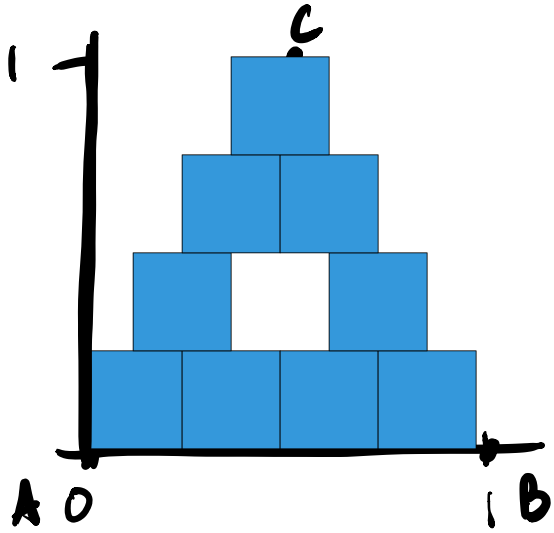
4 steps.

Because we choose which functions randomly, we apply each 1 ⅓ of the time, over time it eventually fills

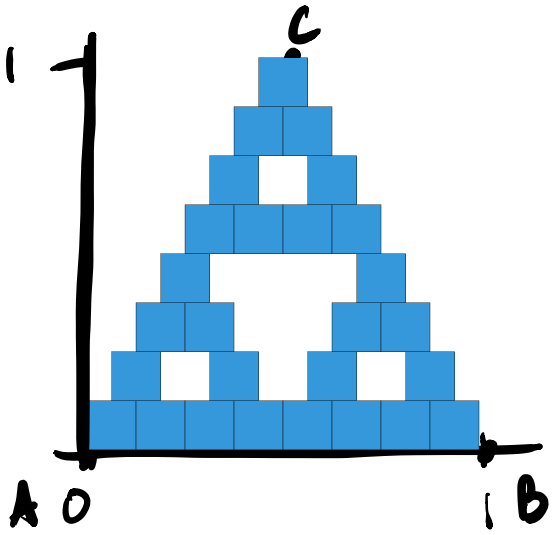


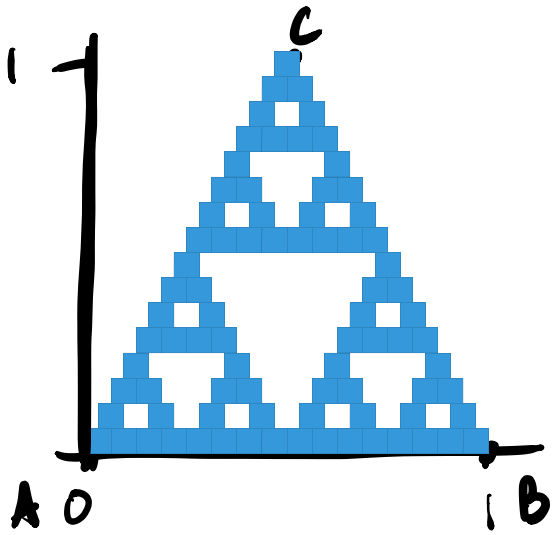


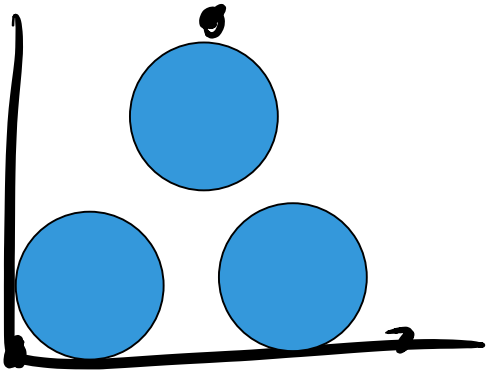
1 step.

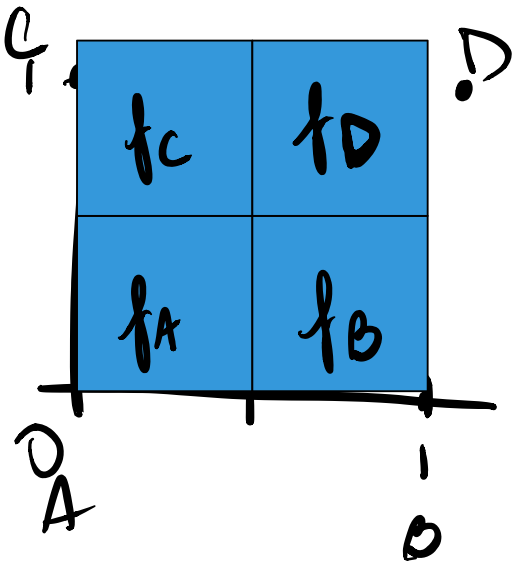


2 steps


C
A
O
B


C
A
O
B





f_A, f_B

f_C, f_D

halfway to the vertex