Agent based models: spatial environments

CSCS 530 - Marisa Eisenberg

Types of ABM environments

- Non-spatial (or at least, not explicitly spatial)
- Network topologies (may be thought of as spatial or not)
    - E.g. subway networks vs. contact networks
- Explicitly spatial environments
    - Grids
    - 2D or 3D space
    - Mapping - have agents move through a real-world-based environment

Grids

- We have seen several examples of this (cellular automata, the Shelling model, forest fire model, etc.)

- A classic example along these lines that uses the environment in an active way is **Sugarscape** (slides mostly borrowed from Lynette Shaw)

Sugarscape Model

- Classic, very well-known model of wealth distribution developed in the mid-1990s

- Presented by Joshua Epstein and Robert Axtell in their classic book, *Growing Artificial Societies*

- Begins with a very simple model then explores an extremely wide-range of substantively interesting variations

- Many more variations have been developed since

Classic Sugarscape: Environment

- Agents exist on a square-lattice known as “Sugarscape” w/ individual lattice positions that generate a generic resource called “sugar”

- **Patch variables**
  - Current sugar level, max sugar capacity

- **Patch methods**
  - Patches regenerate sugar according to some function G_{alpha} where alpha = units of sugar grown back in one time step, up to max capacity

Classic Sugarscape: Agents

- **Agent variables**
  - Position: x,y coordinates on the Sugarscape
  - Sugar level: how much sugar agent currently has (no limit)
  - Metabolism (m): how many units of sugar it “burns” per time step
  - Vision (v): how many lattice positions away an agent can “look” for sugar

Classic Sugarscape: Agent actions

- Movement (M):

    1. Look out v number of positions in NWSE directions (no diagonal!)

    2. Move to nearest, unoccupied position w/the most sugar

    3. Collect all sugar on that position

Classic Sugarscape: Agent actions

- Metabolize:

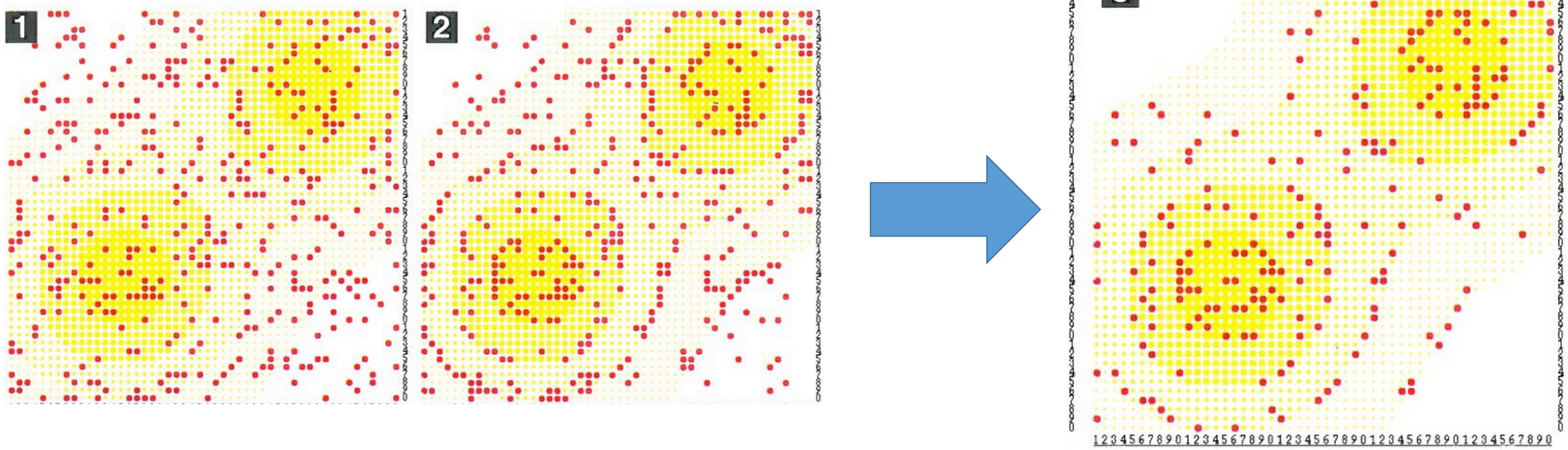
    1. Decrement sugar level by m units

    2. If current sugar level < 0, die

Baseline model for Sugarscape

- Baseline Model:
  - Random initialization of agents for v, m, and initial sugar level
  - Set alpha = infinity

Animation II-1. Societal Evolution under Rules ($\{G_\infty\}$, $\{M\}$) from a Random Initial Distribution of Agents

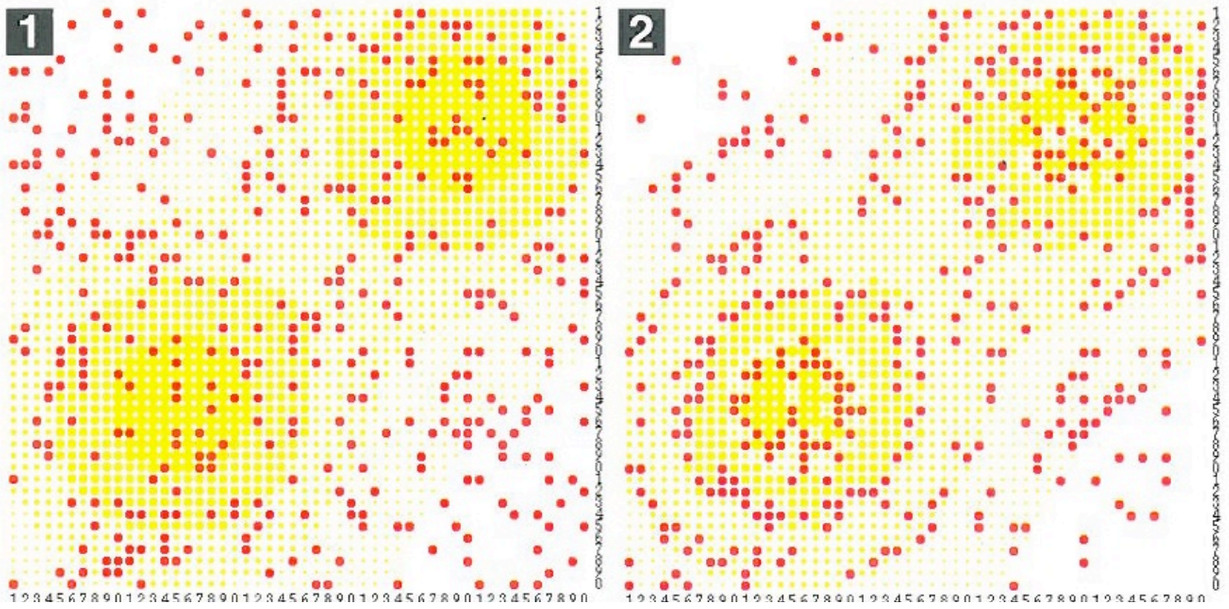


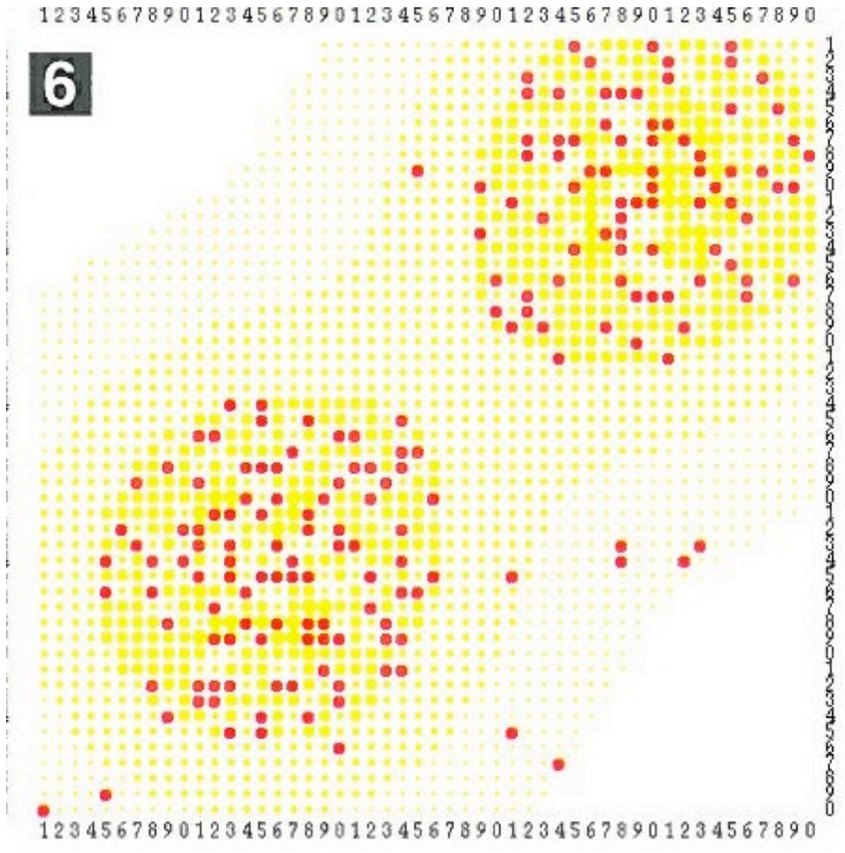
Figures from *Growing Artificial Societies*

Baseline model for Sugarscape

- Baseline Model:
  - Random initialization of agents for v, m, and initial sugar level
  - Set alpha = 1

Animation II-2. Societal Evolution under Rules ($\{G_1\}$, $\{M\}$) from a Random Initial Distribution of Agents





Figures from *Growing Artificial Societies*

Wealth Distributions in Sugarscape

- In baseline model, no replacement for agents that die. Living agents accumulate sugar indefinitely —> no stationary wealth distribution

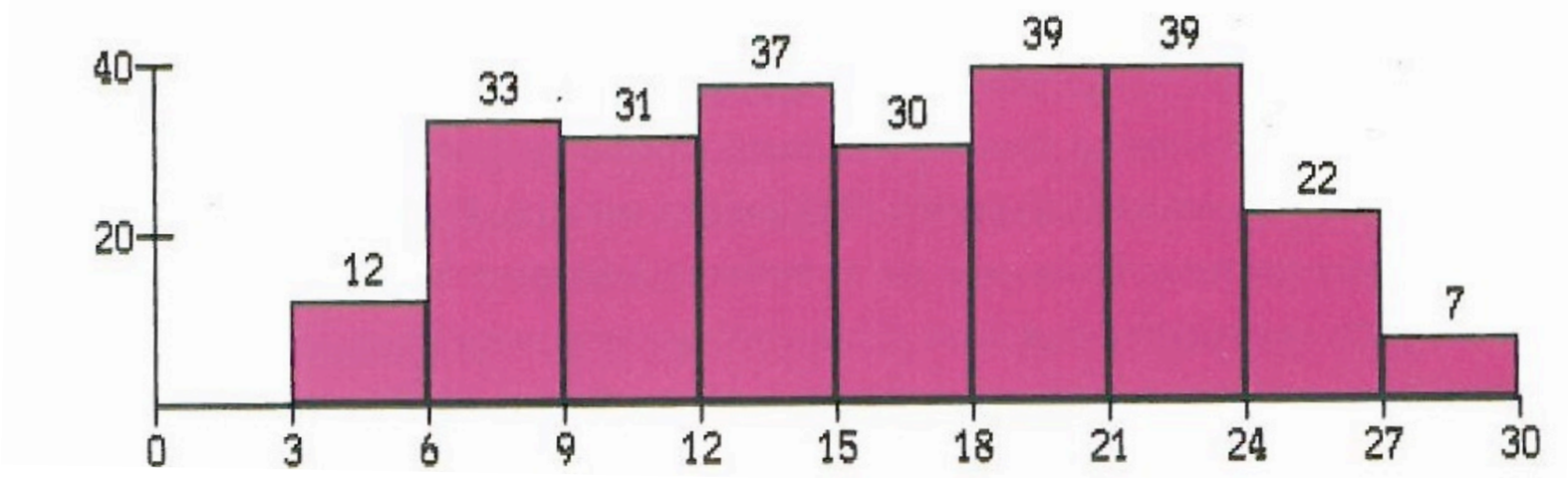
- Variant: Add Replacement Rule

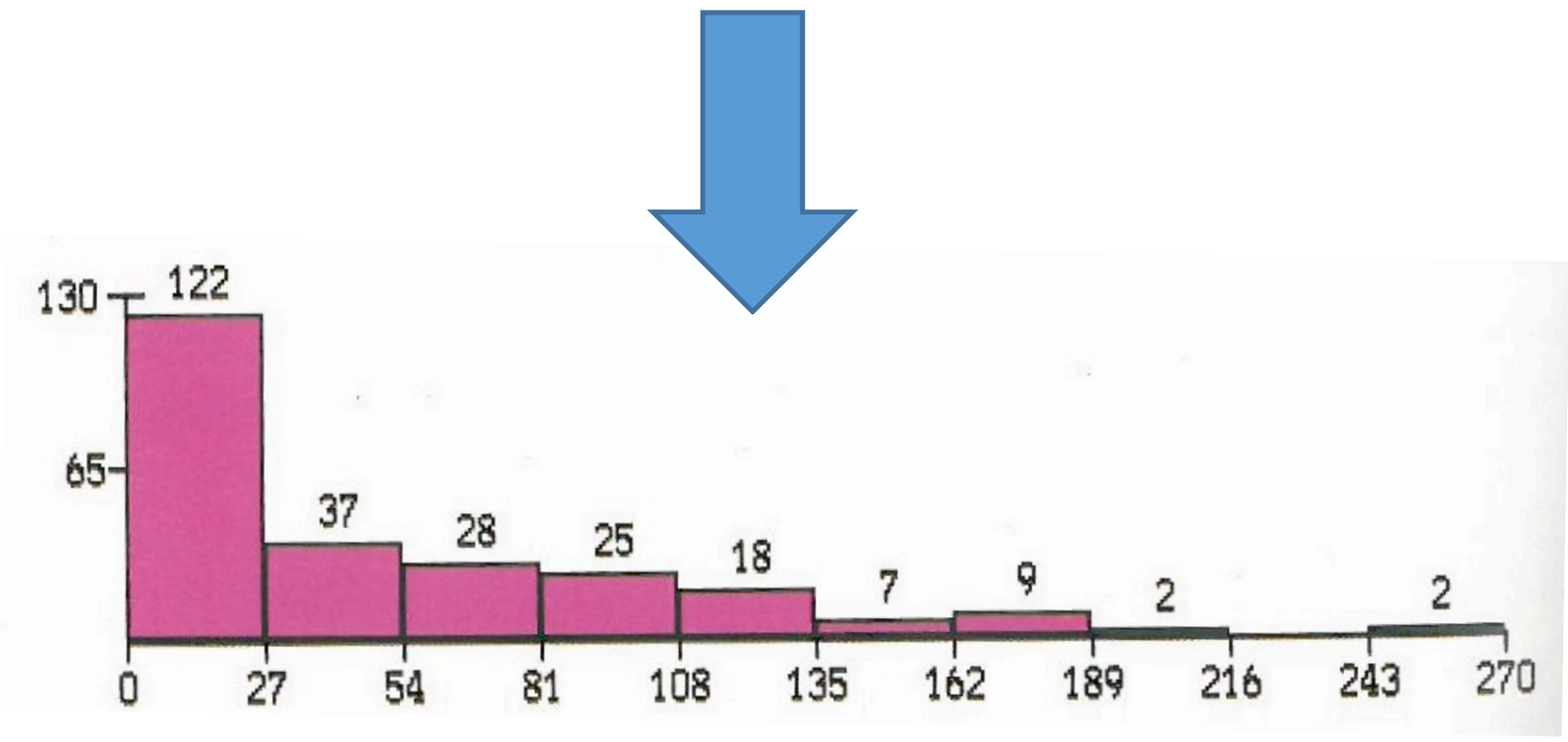
  - Each agent gets a max achievable age drawn from [a,b]. Die after that age (or before if sugar < 0)

  - When agent dies, replace w/a randomly initialized (including position) agent

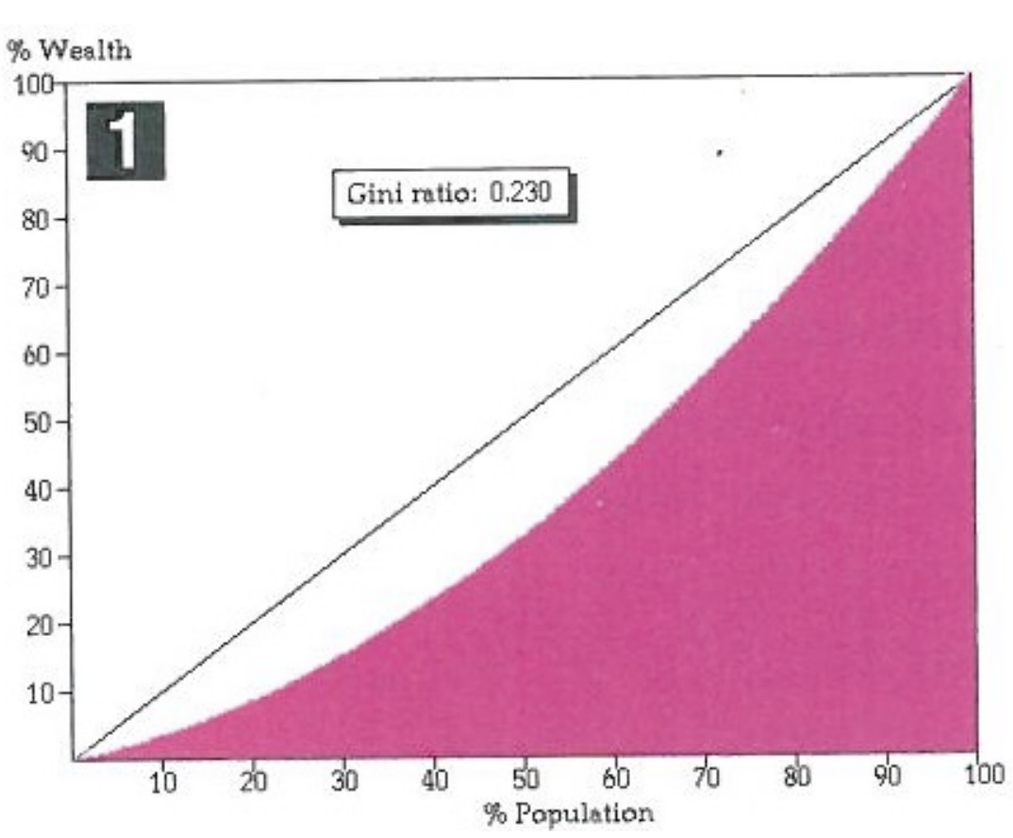
Emergence of a skewed wealth distribution

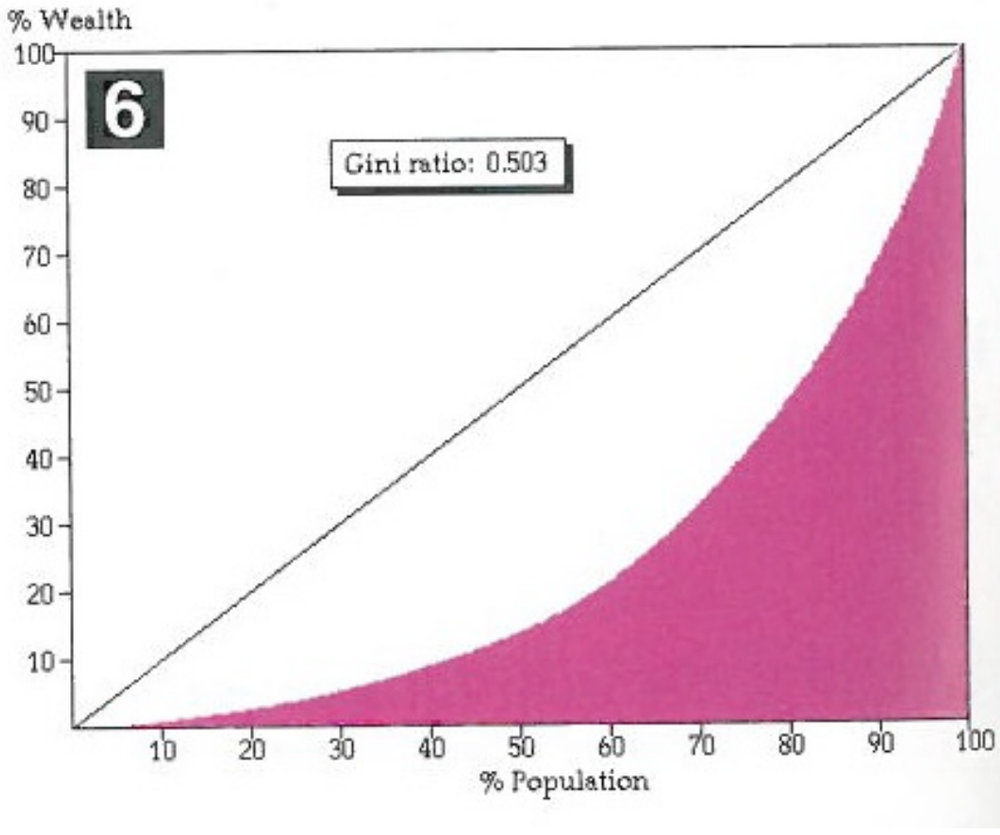
Animation II-3. Wealth Histogram Evolution under Rules ({$\mathbf{G}_1$}, {**M**, $\mathbf{R}_{[60,100]}$}) from a Random Initial Distribution of Agents





Emergence of inequality





Demonstrates how empirically reminiscent patterns of inequality can emerge from a set of simple rules + environmental and actor heterogeneity

Migration variations

- Alter initial random distribution of agents to add more “structure” in starting position

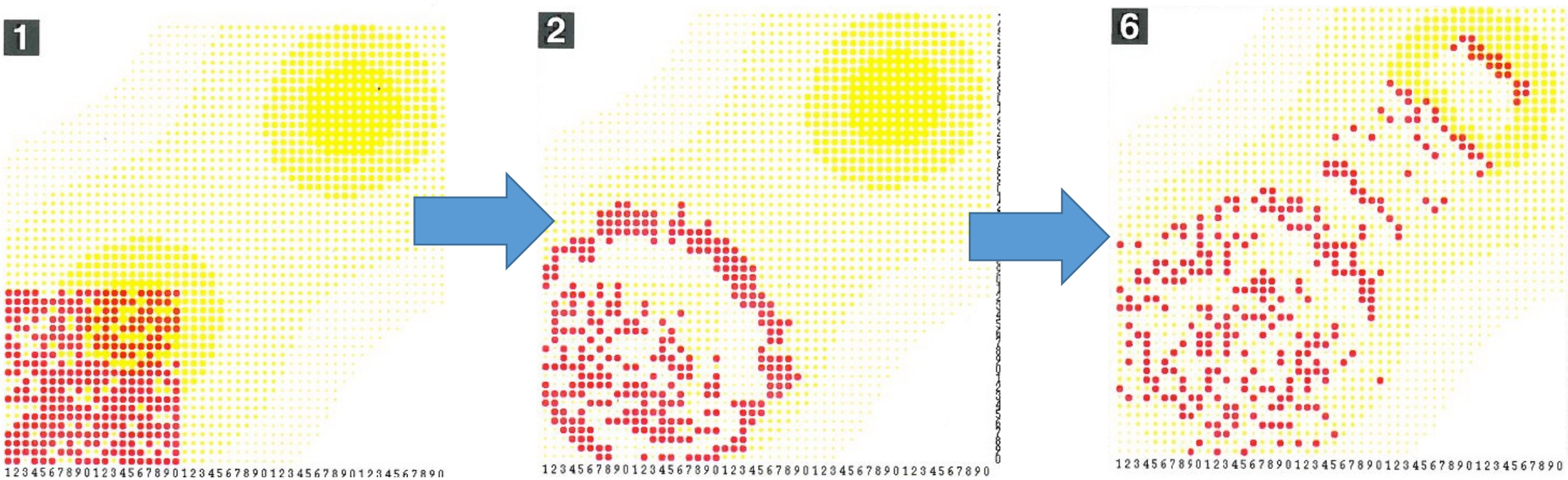
- Seasonal Migration

  - Introduce spatial and temporal patterning in alpha by creating an “equator” in space and “seasons” of higher-lower alphas in the two regions

  - Note: only environmental changes. No changes to movement/agent rules

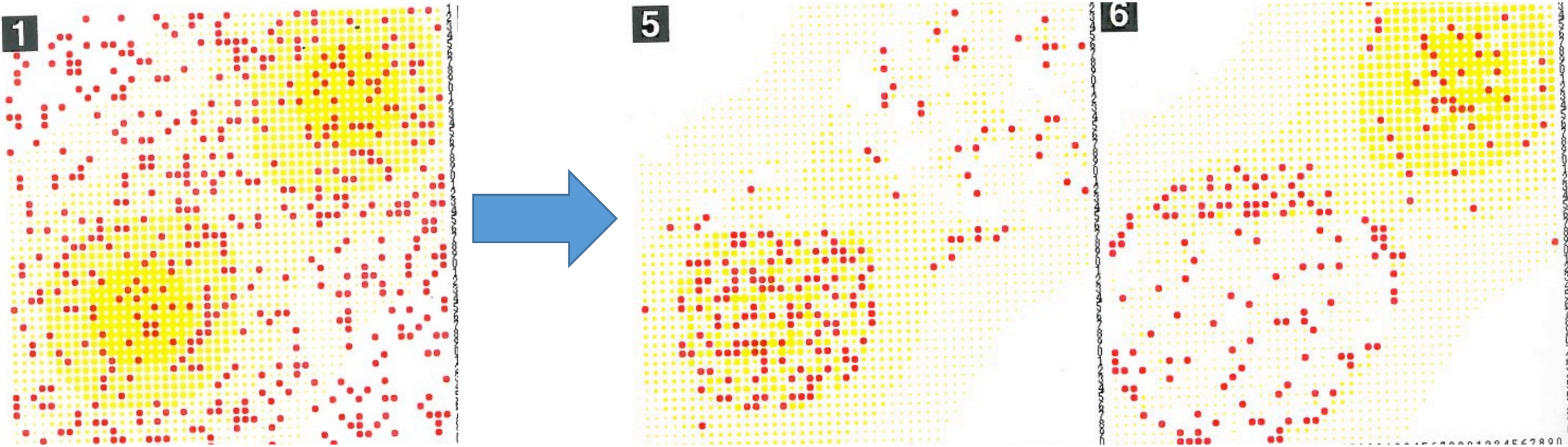
Migration

- Diagonal pattern of movement



Seasonal Migrations

- Migration + emergence of “hibernators” and “migrators”



The World of Sugarscape Elaborations

- Though the baseline model is extremely simple, a wide number of elaborations and variations have been developed to explore a host of other issues
  - Social networks
  - Sexual reproduction
  - Cultural change
  - War and conflict
  - Inheritance and wealth
  - Disease

Sugar and Spice: a Market Dynamics Elaboration

- This elaboration begins with the introduction of a second resource to the environment, “spice”

- Agents now have both a sugar level and a spice level with a corresponding *m* for each. Die if either level < 0

- Movement now changes to be driven by a “Welfare” (W) function

Sugar and Spice: Agent Welfare Function

- 1 = Sugar, 2 = Spice

- $m_T = m_1 + m_2.$

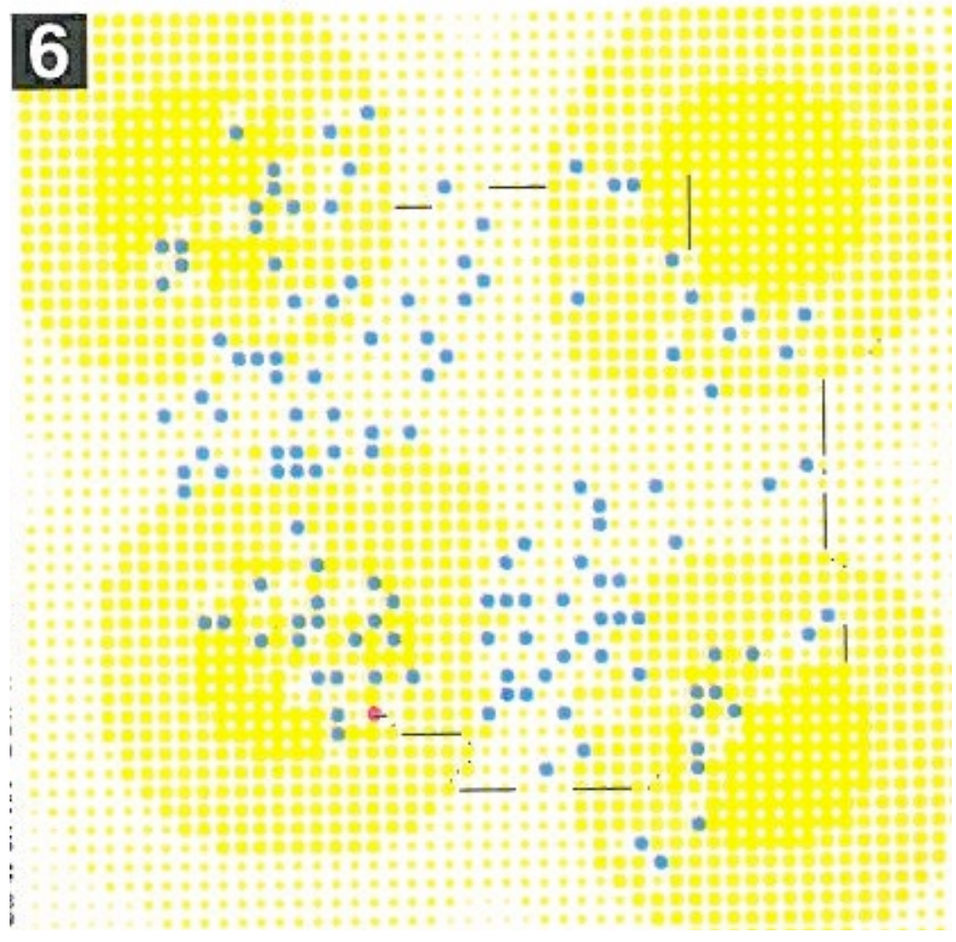
  $W(w_1, w_2) = w_1^{m_1/m_T} w_2^{m_2/m_T},$

- Movement Rule Change:

- Replace “unoccupied position with maximum sugar level” with “unoccupied position maximum welfare increase”

Sugar and Spice: No Trade

- Oscillating movements between Sugar and Spice piles
- Lower carrying capacity than 1 commodity scenario



Sugar and Spice: Trade Rules

- With 2 commodities, can now allow for decentralized trade between agents
- Marginal Rate of Substitution (MRS) $\frac{\frac{w_2}{m_2}}{\frac{w_1}{m_1}}$

	$MRS_A > MRS_B$		$MRS_A < MRS_B$	
Action	A	B	A	B
Buys	sugar	spice	spice	sugar
Sells	spice	sugar	sugar	spice

Sugarscape

Agent trade rule **T**:

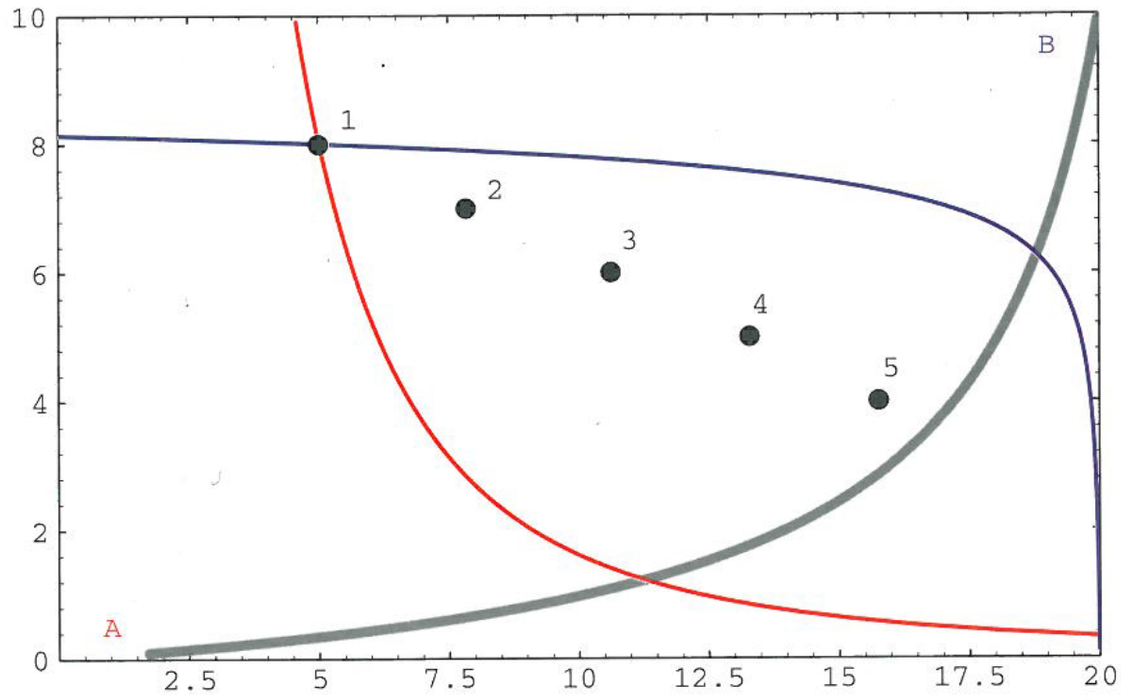
- Agent and neighbor compute their *MRS*s; if these are equal then end, else continue;
- The direction of exchange is as follows: spice flows from the agent with the higher *MRS* to the agent with the lower *MRS* while sugar goes in the opposite direction;
- The geometric mean of the two *MRS*s is calculated—this will serve as the price, p;
- The quantities to be exchanged are as follows: if $p > 1$ then p units of spice for 1 unit of sugar; if $p < 1$ then $1/p$ units of sugar for 1 unit of spice;
- If this trade will (a) make both agents better off (increases the welfare of both agents), and (b) not cause the agents' *MRS*s to cross over one another, then the trade is made and return to start, else end.

Sugarscape

- Local Pareto Optimality

- Can show that these rules for exchange and price formation, played out multiple times in a bargaining dyad, achieves a local Pareto optimum

Figure IV-2. Edgeworth Box Representation of Two Agents Trading according to Rule **T**



Sugarscape

- Decentralized trading can lead to a stable, average trade price w/o the need for a central “auctioneer”
- Also, increases carrying capacity of system

Figure IV-3. Typical Time Series for Average Trade Price under Rule System ({$\mathbf{G}_1$}, {**M**, **T**})

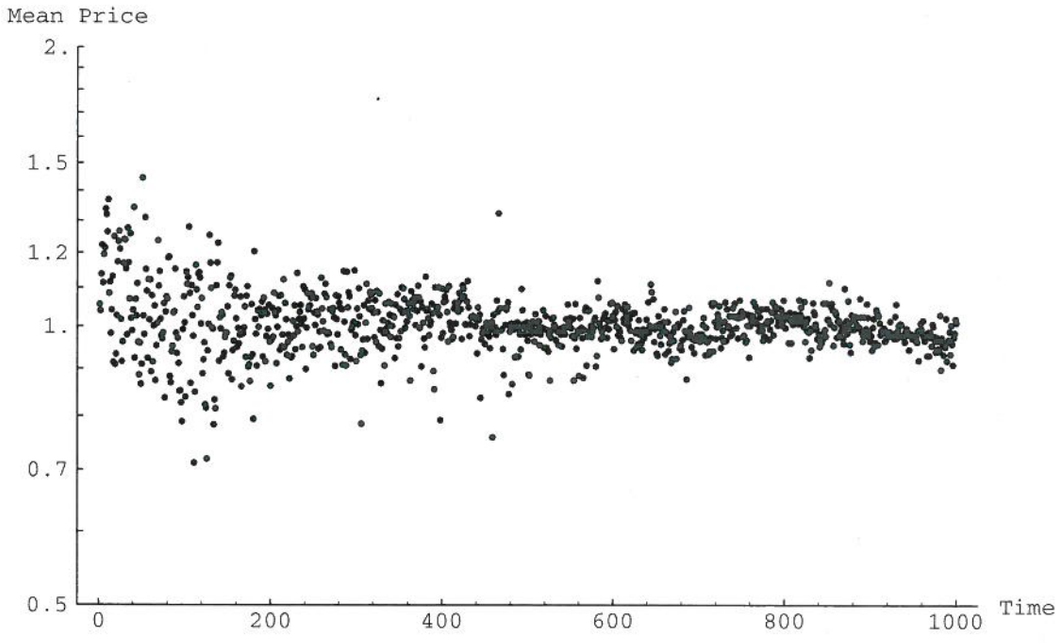
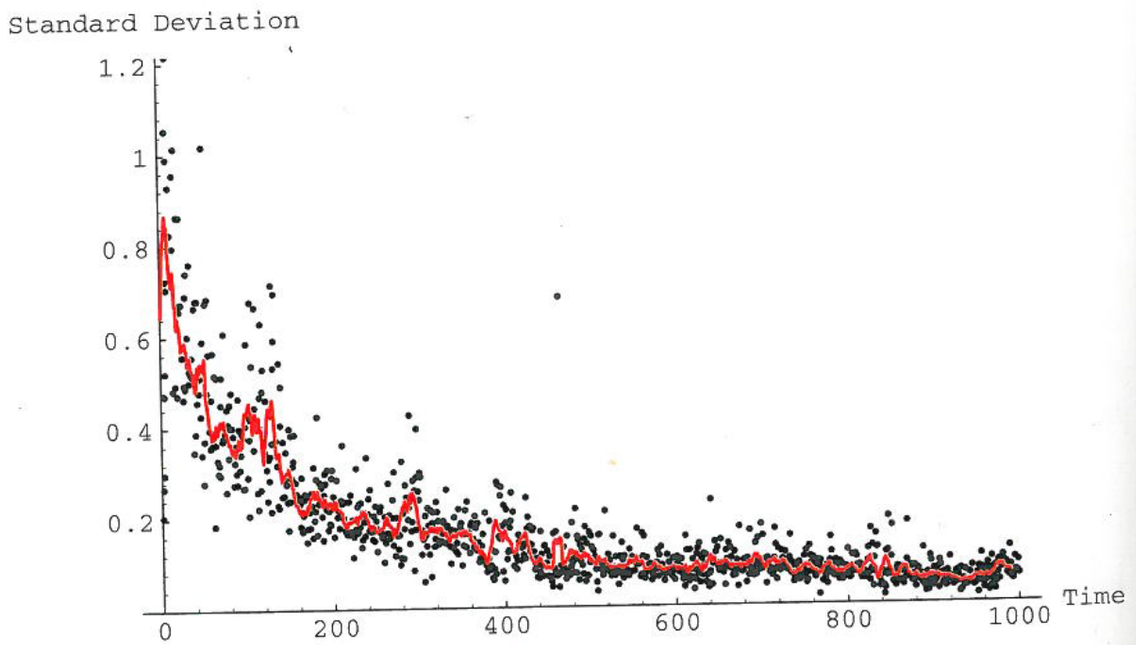


Figure IV-5. Typical Time Series for the Standard Deviation in the Logarithm of Average Trade Price under Rule System ({$\mathbf{G}_1$}, {**M**, **T**})



Sugarscape extensions

- Horizontal inequality
- Ability to get into “Far From Equilibrium Economics”
- Price variance strongly impacted by agent vision
- Local efficiency, Global inefficiency

2D or 3D space

- 2D space
  - Abstract 2D spaces with arbitrary setups but not on an explicit grid (e.g. movement is decided based on a distribution not on a grid)
  - Imaging data (e.g. MRI, microscopy imaging)
  - Map data
- 3D space
  - Often used for flocking simulations (e.g. Boids)
  - Also for modeling complex biological or physical domains (e.g. cellular environments, etc.)

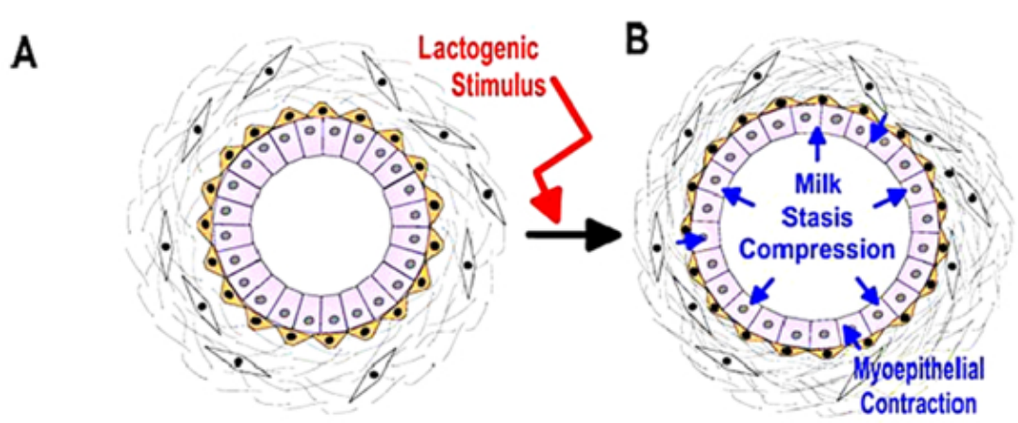
Abstract 2D & 3D space

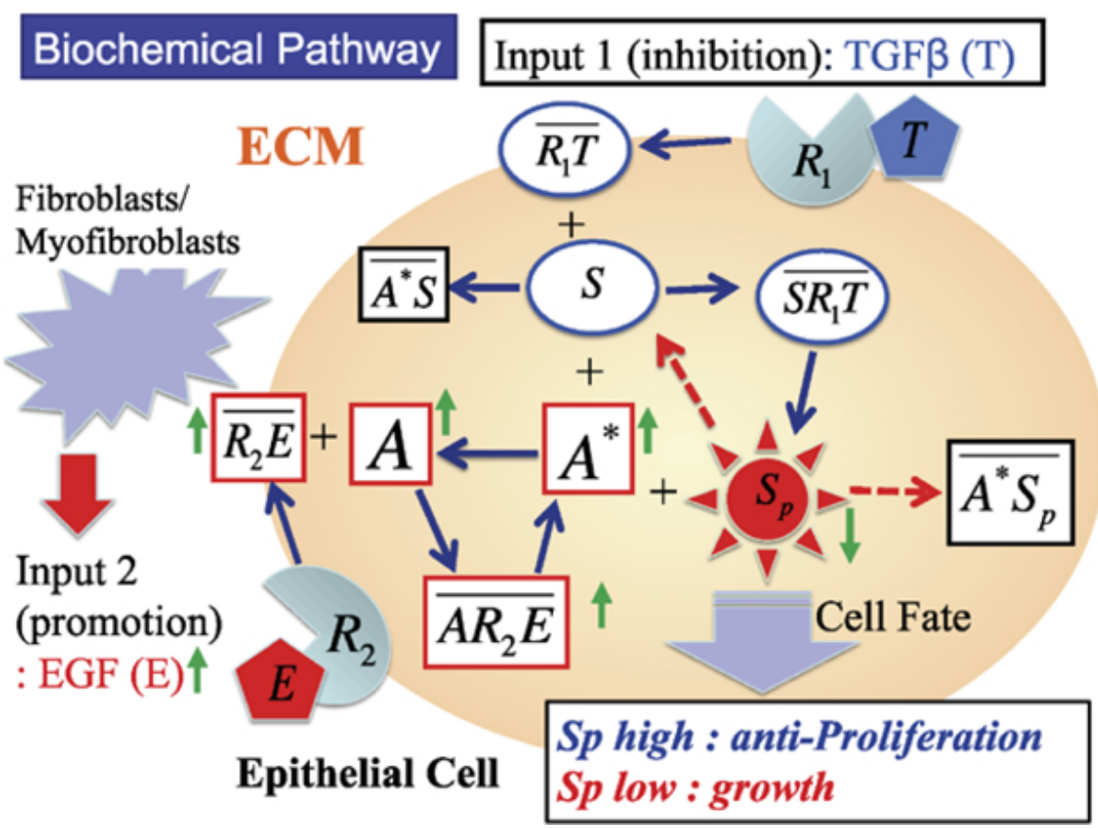
- We have worked with several examples of this in 2D space (e.g. ants, rabbits/foxes)

- Complex environments can be generated

Example: hybrid model of breast cancer dynamics

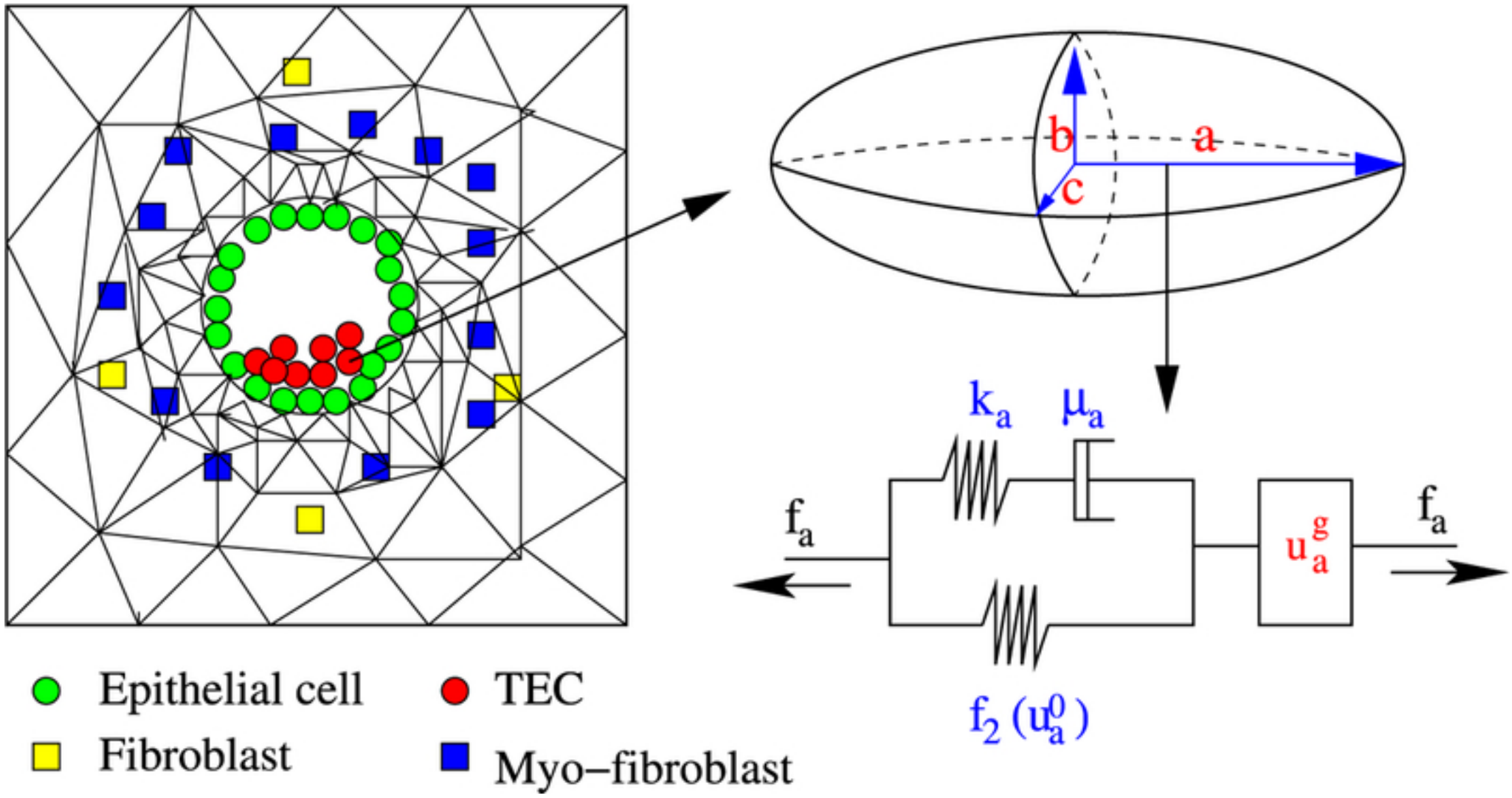
- Transformation from normal ducts to solid tumor in breast cancer





Kim & Friedman - https://www.ncbi.nlm.nih.gov/pubmed/23292359

Cells as agents within the duct



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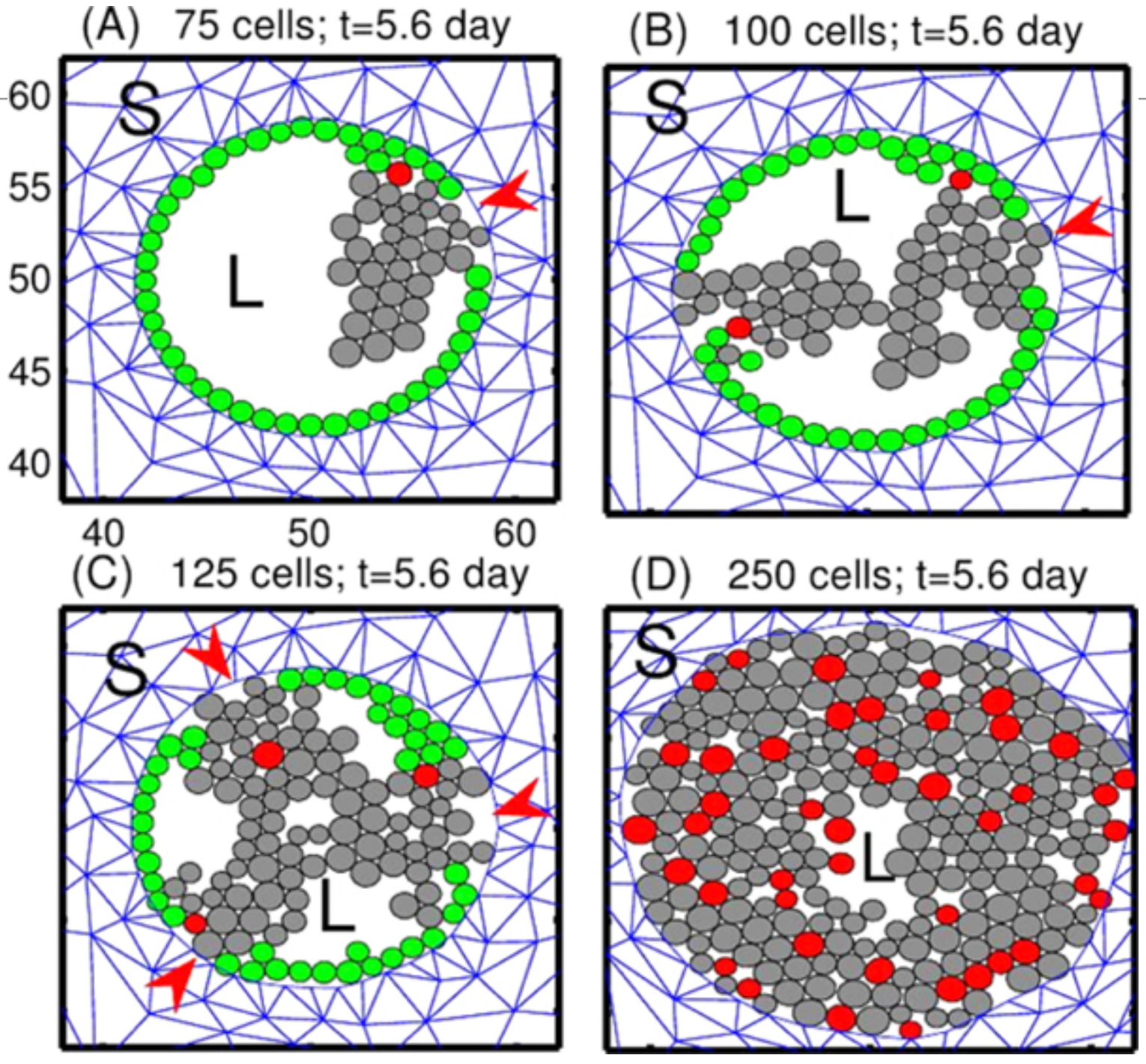

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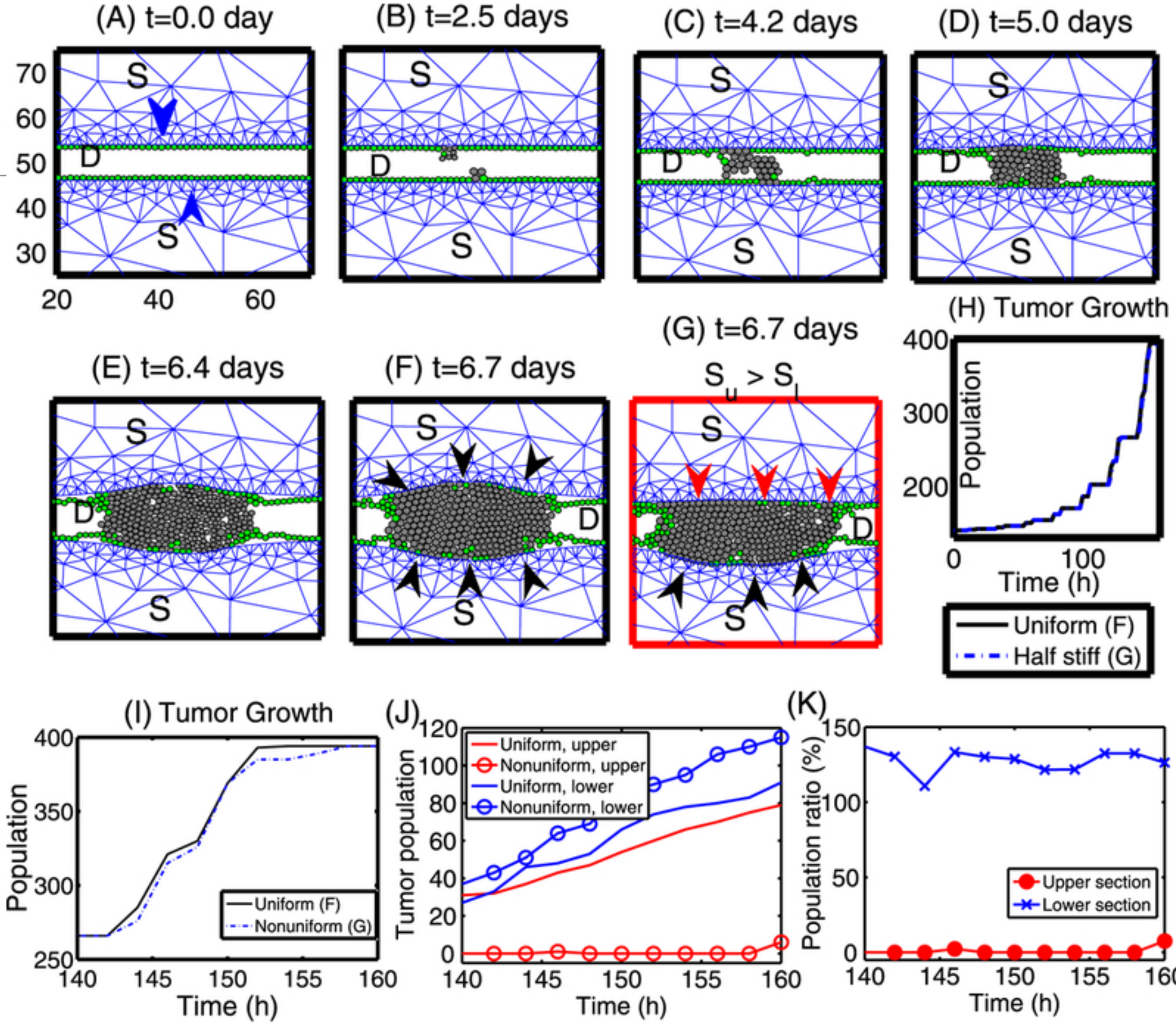
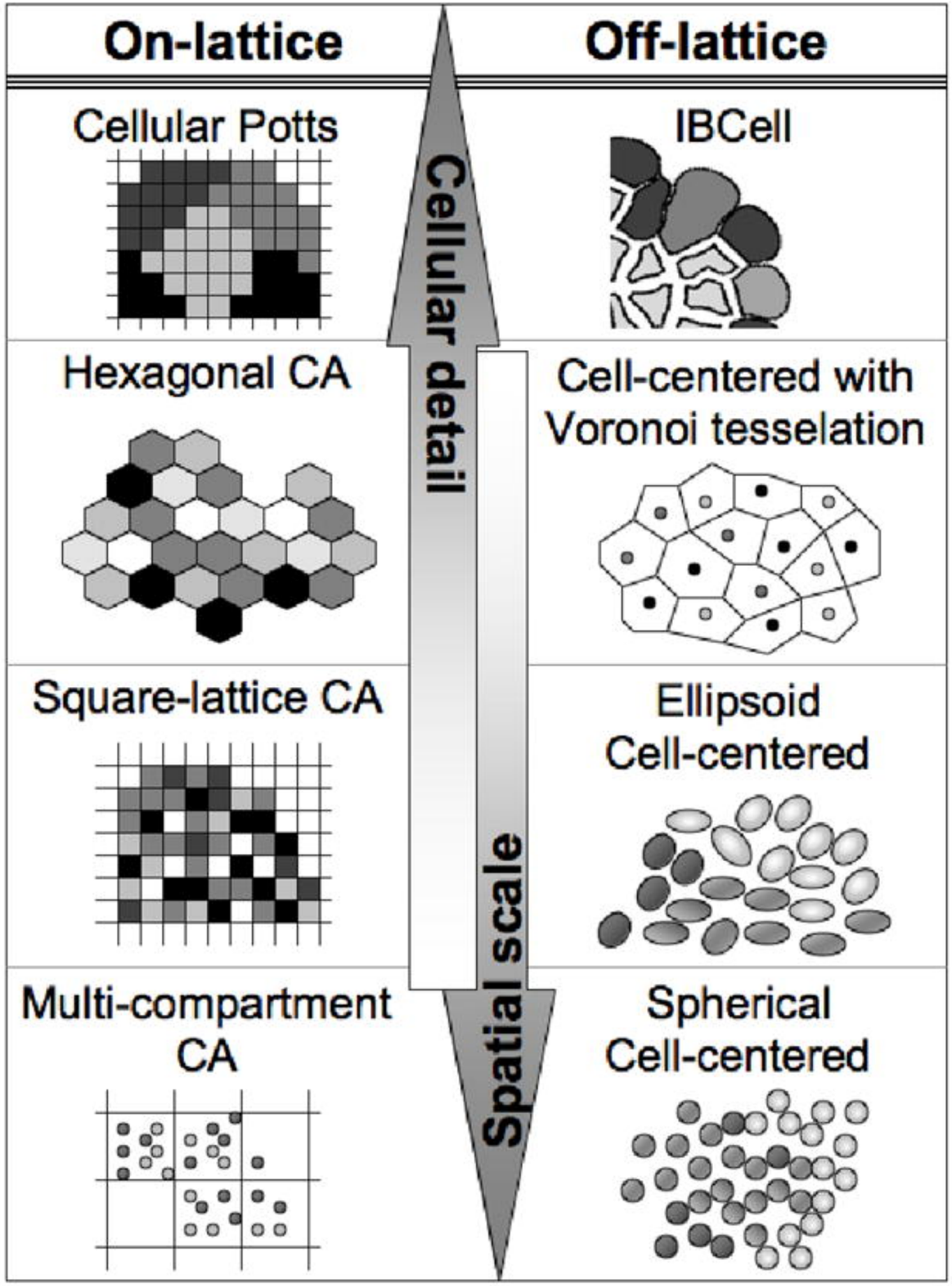
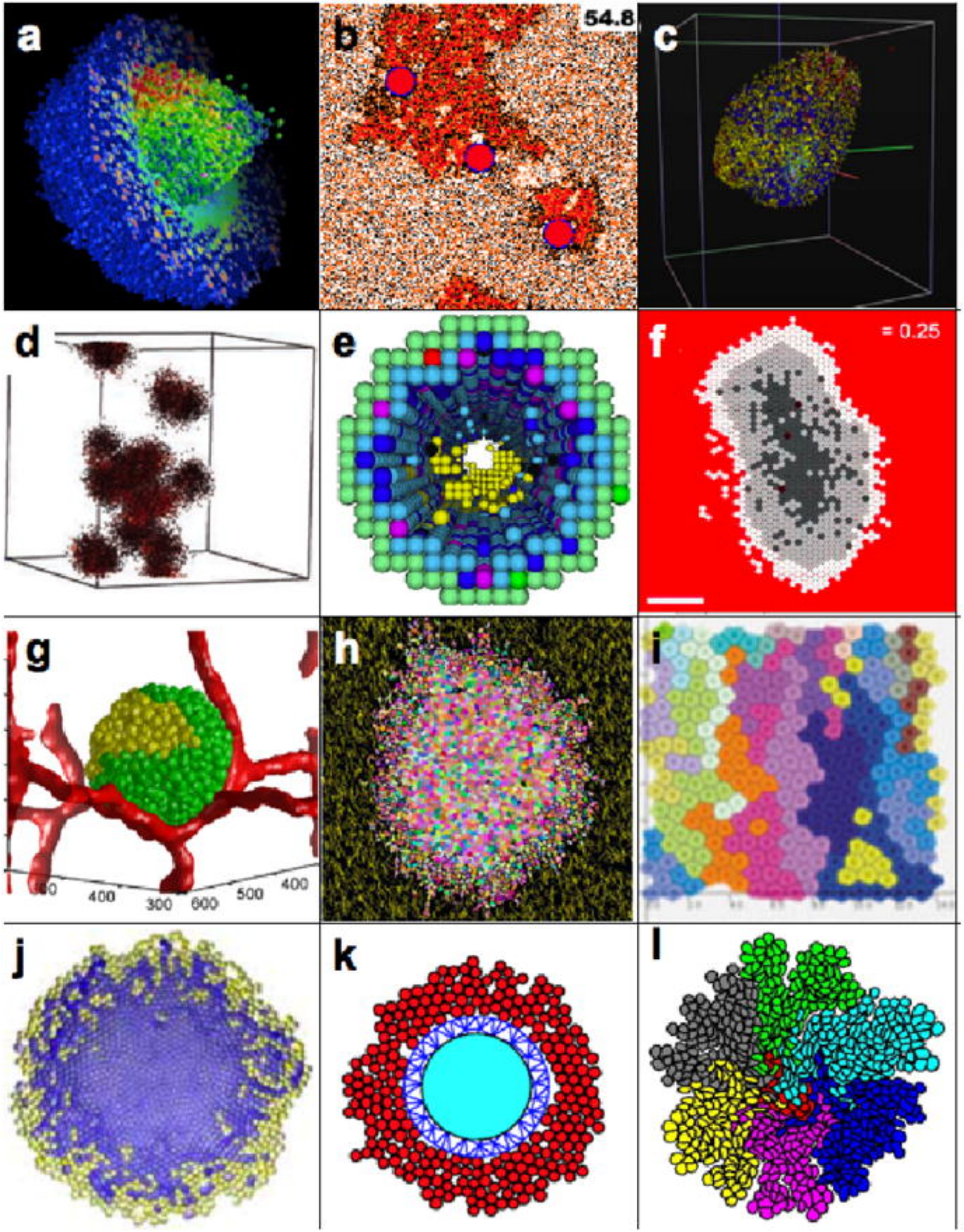

(A) t=0.0 day
(B) t=2.5 days
(C) t=4.2 days
(D) t=5.0 days
(E) t=6.4 days
(F) t=6.7 days
(G) t=6.7 days
S
D
S
$S_u > S_l$
70
60
50
40
30
20
40
60
(H) Tumor Growth
Population
400
300
200
0
100
Time (h)
Uniform (F)
Half stiff (G)
(I) Tumor Growth
Population
400
350
300
250
140
145
150
155
160
Time (h)
Uniform (F)
Nonuniform (G)
(J)
Tumor population
120
100
80
60
40
20
0
Uniform, upper
Nonuniform, upper
Uniform, lower
Nonuniform, lower
(K)
Population ratio (%)
150
100
50
0
Upper section
Lower section



Rejniak & Anderson - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3057876/


a
b
54.8
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e
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= 0.25
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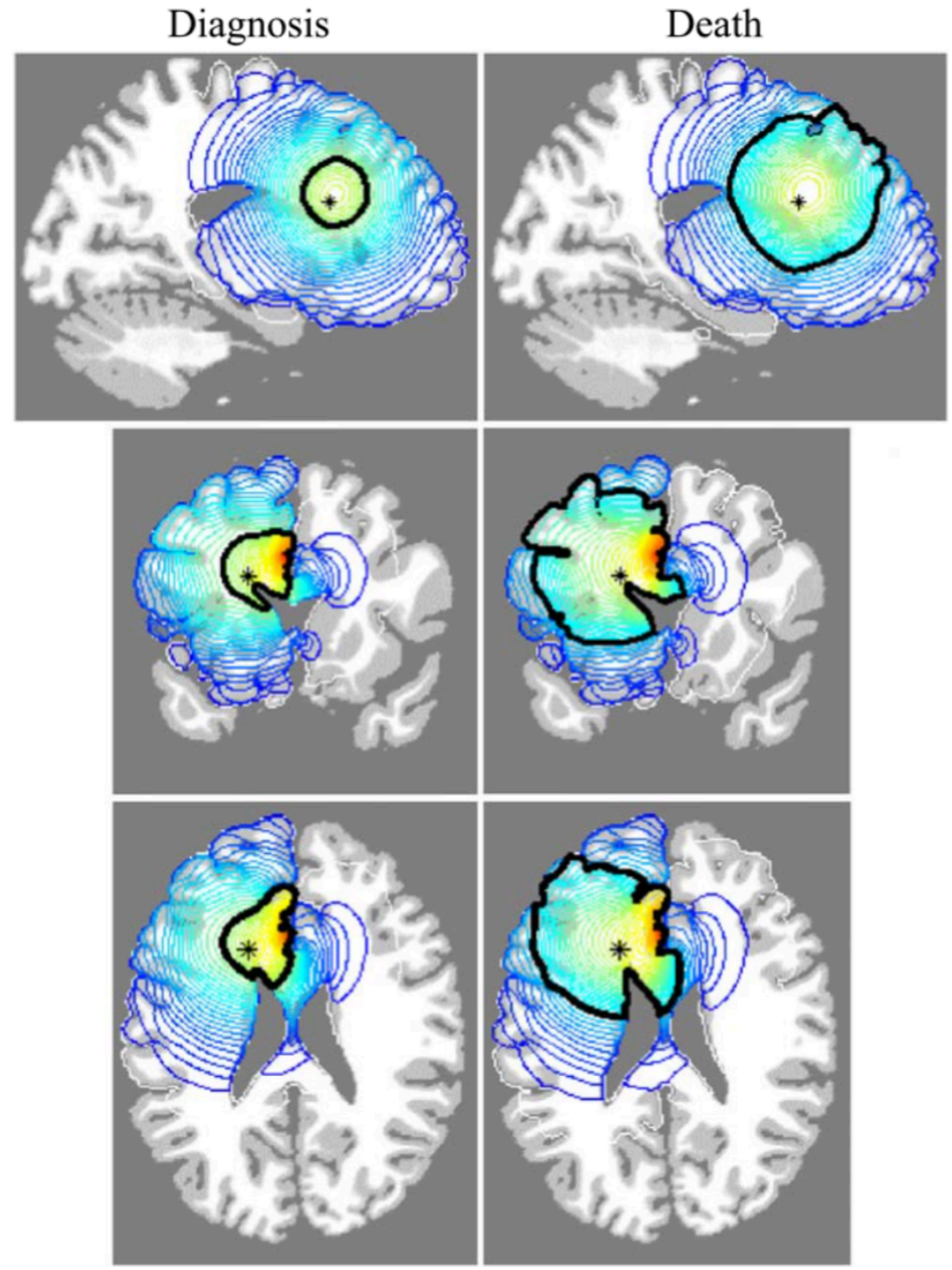
Imaging data

- Exists at many scales—landscapes to cells
- Often requires some image processing to identify regions with key features for the agents to interact with
- We've seen another example of this in the model of the Ancestral Puebloan communities that we looked at previously



Swanson et al. 2003 Virtual and real brain tumors: using mathematical modeling to quantify glioma growth and invasion

Mapping data

- Often used for modeling commuting patterns, disease spread, social dynamics, etc.

- Modeling with mapping is similar to other ABM with space, just using the map to determine where features are

- Example: FRED synthetic population model

- https://fred.publichealth.pitt.edu/measles