Lecture 8: Dynamics on and of Networks

Complex Systems 530
2/11/20 & 2/18/20

How to generate networks?

- Real world networks (static or dynamic)—lots of network data out there
- Random networks!
  - Many of these can be used either as
    - static networks to run dynamics *on*, or
    - models of dynamics *of* networks

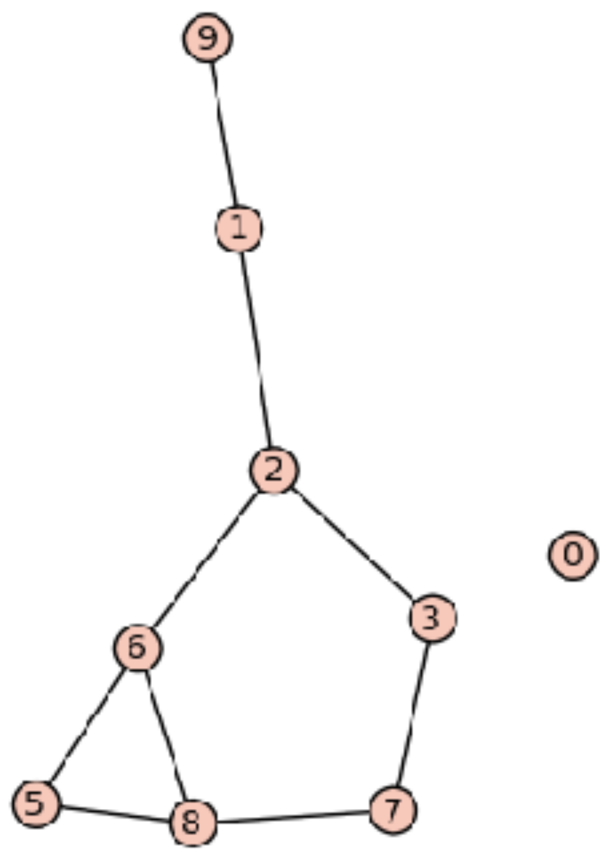
Random Networks

- Why would you want to do this?
  - Often want to simulate network formation or simulate dynamics on networks
  - May not know exact network
  - But often do know some general features of the network (e.g. degree distribution)
  - So: simulate random networks with those features

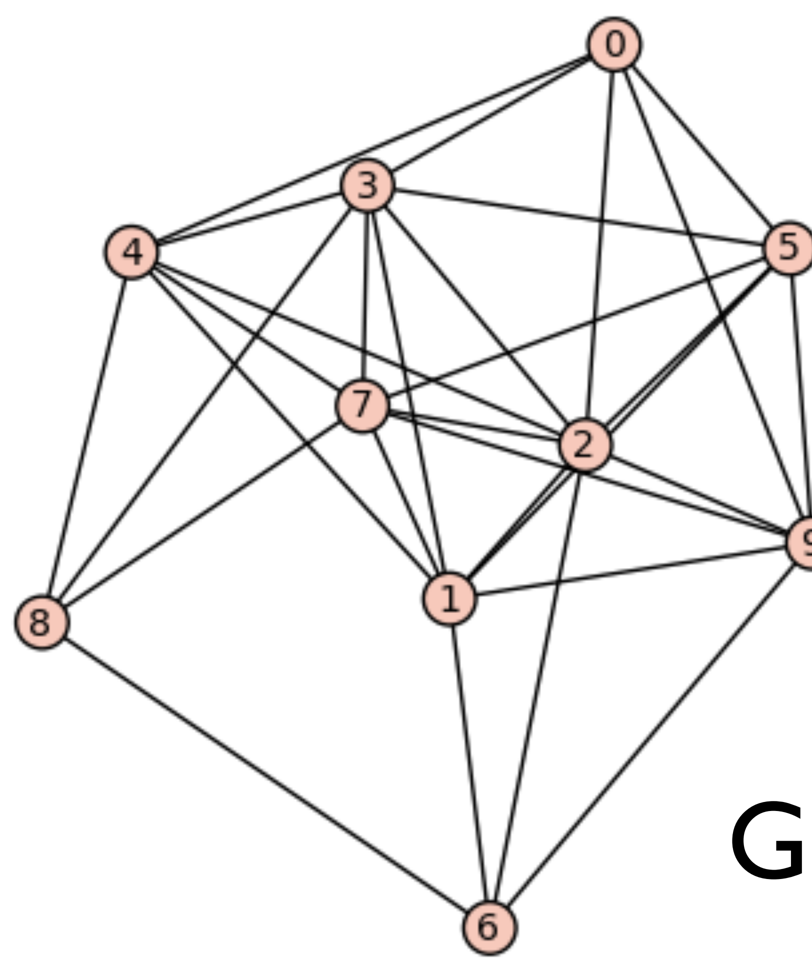
Erdös-Rényi Networks

- **Erdös–Rényi** (also Gilbert) Network - two forms:
  - *G(n,p)* - network on *n* nodes with each edge having probability *p* of existing
  - *G(n,M)* - network on *n* nodes with *M* edges chosen randomly
  - **Often called a “random graph”** even though all of the networks here are also random

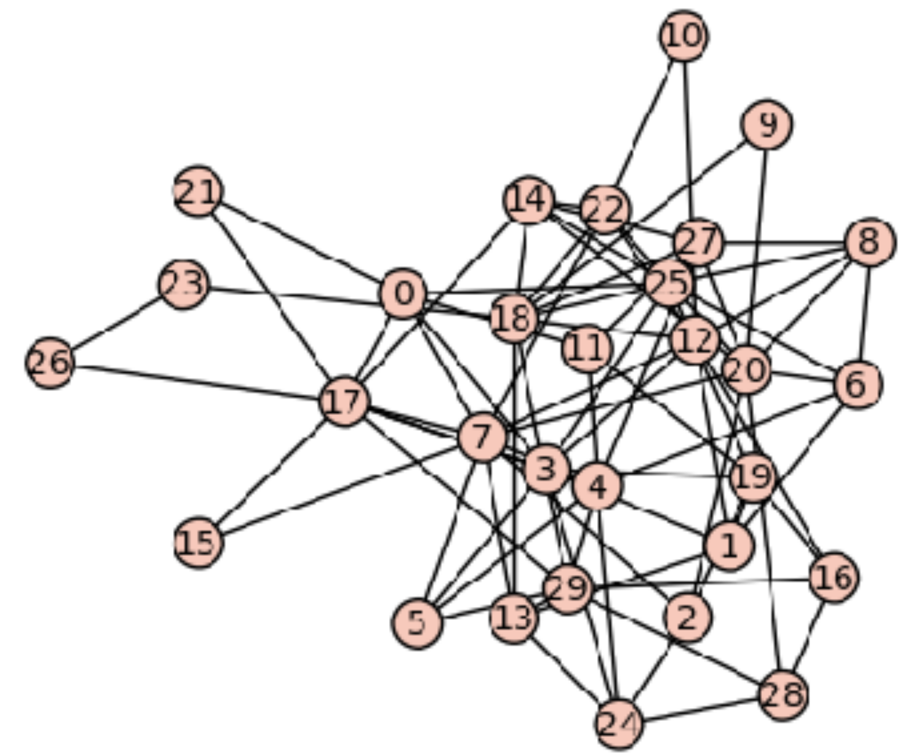
G(n,p)



G(10,0.2)

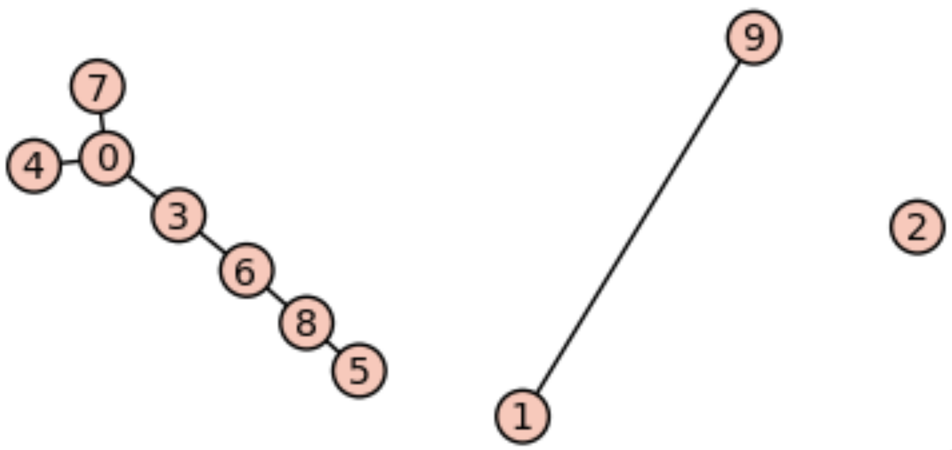


G(10,0.6)

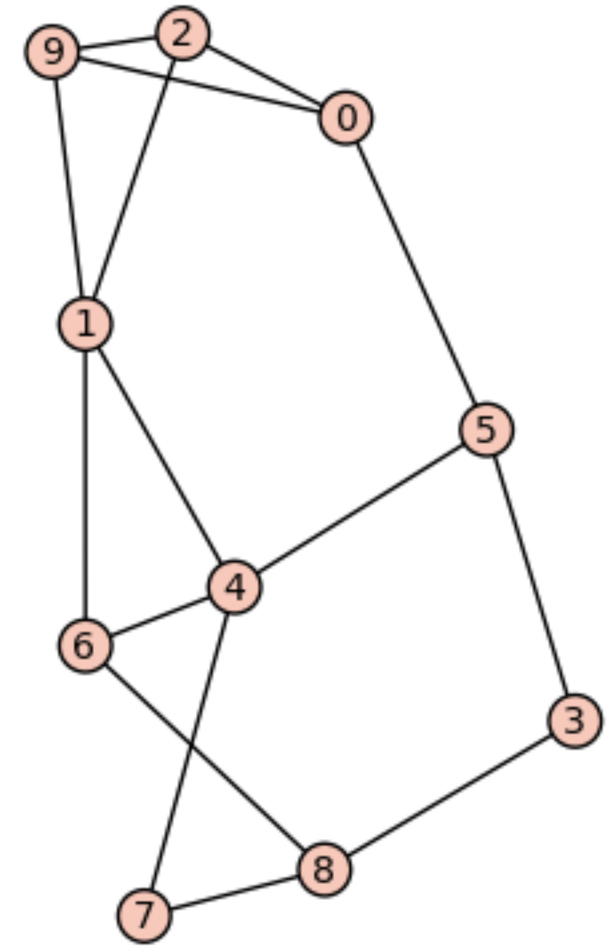


G(30,0.3)

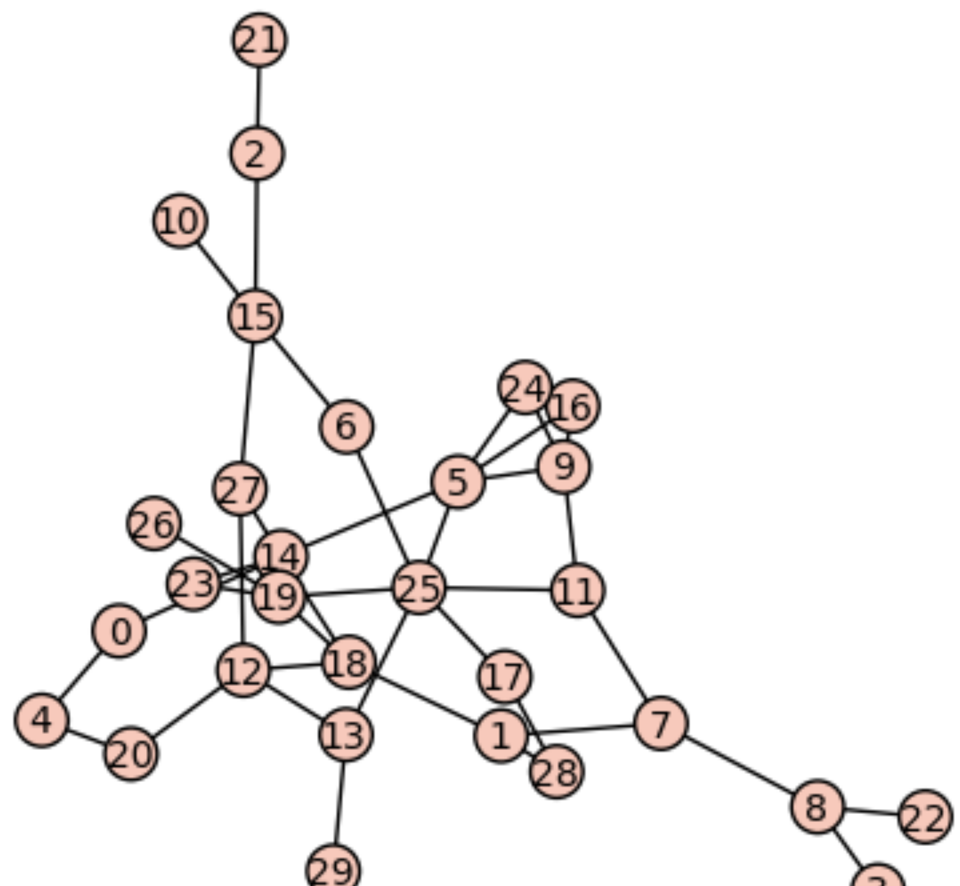
G(n,M)



G(10,7)



G(10,15)



G(30,40)

Erdös-Rényi Networks

- Not so realistic for lots of things (e.g. social networks, many gene/protein/biological networks)
- But, often handy as a test case/comparison point (e.g. if evaluating whether a mean-field model is a reasonable approximation)
- Useful for making analogs of homogeneous mixing (e.g. from SIR or compartmental models)

Erdös-Rényi Networks

- Let you sample from the full space of possible graphs with minimal assumptions
- If a property of a network is reproduced by ER, may suggest it's not a special feature of the network driving it—alternatively if ER does not reproduce this property, it may be more "interesting"

Erdös-Rényi Networks

- Lots of mathematical theory for random matrices (e.g. useful for examining adjacency matrices) and random graphs, particularly for Erdös-Renyi graphs, e.g.
    - Degree distribution, giant component, etc.

Milgram's Small World Experiment

- Sent packages to random people in Wichita, Kansas

- Letter inside asked them to forward to a target person in Sharon, Massachusetts

- Told they could mail the letter directly to the target person only if they knew him personally, otherwise send it & instructions to a relative or friend they thought would be more likely to know the target person

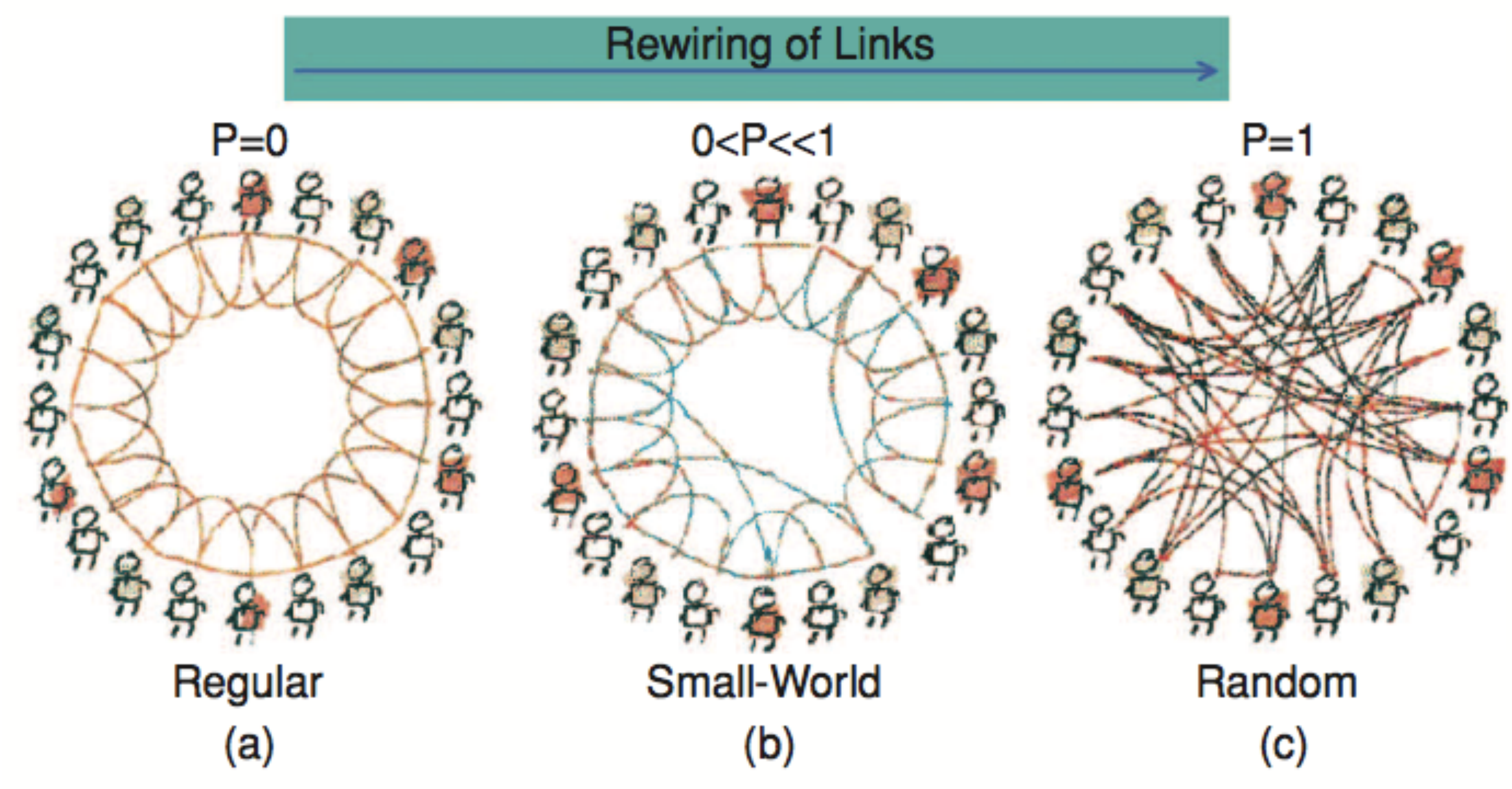
Milgram's Small World Experiment

- Many letters didn't make it, but among those that did, average path length was 6
- "Six degrees of separation"
- How to generate a small world network?

Small World Networks

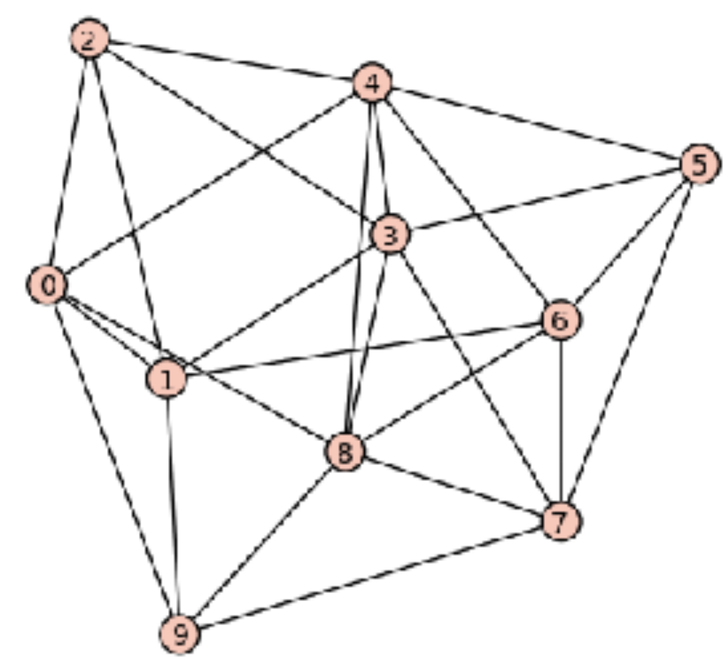
- Regular graphs: clustered, but path length *L* grows linearly with number of nodes *n*
- Erdös-Rényi graphs: not clustered but small path length (grows as *log n*)
- Want to combine both

Newman-Watts-Strogatz Algorithm



Small World Networks

- Most nodes are not neighbors of one another, but most nodes can be reached from every other by a small number of hops or steps
- Average distance *L* between two nodes is proportional to *log n* (where *n* is the number of nodes)



Small World Network

- Creates the “what a small world!” effect: two nodes will tend to have a mutual friend (adjacent node)
- Can be similar to scale free in that can produce hubs as well as sparsely connected individuals
  - Network can be both small-world and scale-free
  - However, N-W-S tends to produce more similar degrees for nodes rather than scale free

Preferential Attachment Networks

- **Barabasi-Albert** algorithm

- Add new nodes to the network sequentially, preferentially connecting them to high-degree nodes

$$p(i) = \frac{\deg(i)}{\sum_j \deg(j)}$$

- Generates scale free networks

Preferential Attachment

- “Rich get richer” (Matthew effect) dynamics make hubs
- Can also implement as a growth process from an existing network

Configuration Models

- Given a degree sequence, generate random network with that sequence

- Random graphs, but with the advantage that the degree sequence can be chosen realistically

- Algorithm: generate ‘stubs’ with the correct degree, then connect pairs of stubs

Configuration Models

- Provides a way to generate random networks consistent with a real-world degree sequence/distribution
  - Often have non-network data that tells us about degree (egocentric data)
  - Or may want to explore the space of graphs that are ‘similar’ to a known network

Dynamics **on** Networks

- Dynamics on nodes and/or edges?
- What variables to consider?
    - Discrete vs. continuous variables
    - Deterministic vs. stochastic

Dynamics **on** Networks

- How to update?
  - Discrete vs. continuous time
  - Synchronous, asynchronous, continuous

Dynamics **on** Networks

- Discrete variable, discrete time—similar to CA! Just a different set of neighbors
    - Implementation is very similar
    - CA models are network models! Using a regular graph with a lattice structure with degree 4 or 8

Example: infectious transmission on a network

- Infectious diseases, information/idea/culture propagation, behavioral dynamics (e.g. transmission of alcohol use behaviors)

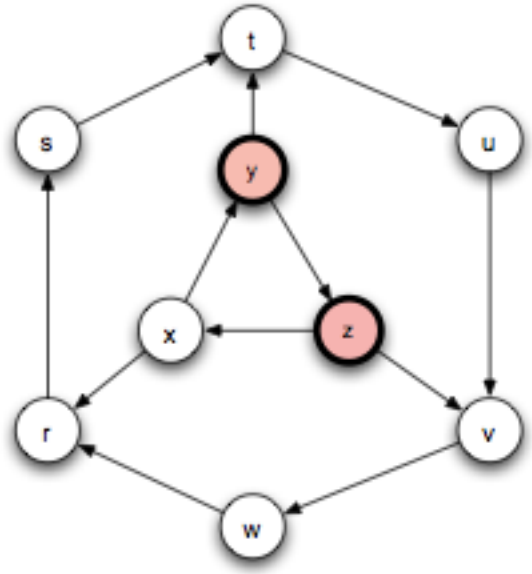
- Nodes may be individuals, or they can be communities

- Edges indicate contact between individuals or communities, or potentially movement between communities

Example: infectious transmission on a network

- Each node may be assigned a status (susceptible/infectious/recovered)
- Or a vector/number (number of infected in that node, numbers of S/I/R in that node)
    - E.g. run an SIR model in each node but allow transmission within-node or between-node

Individual-level network models of disease transmission



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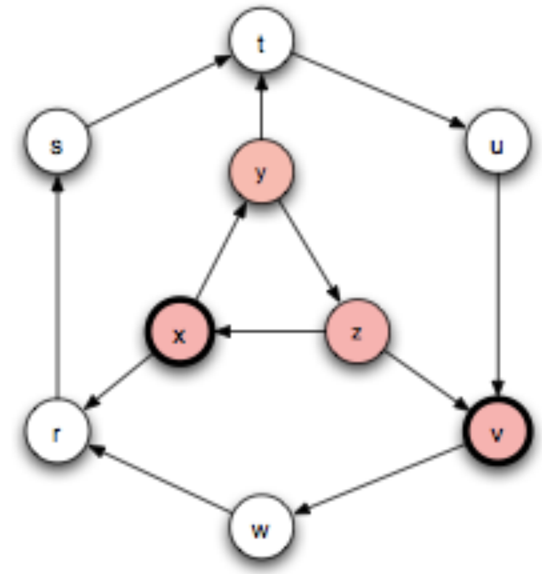

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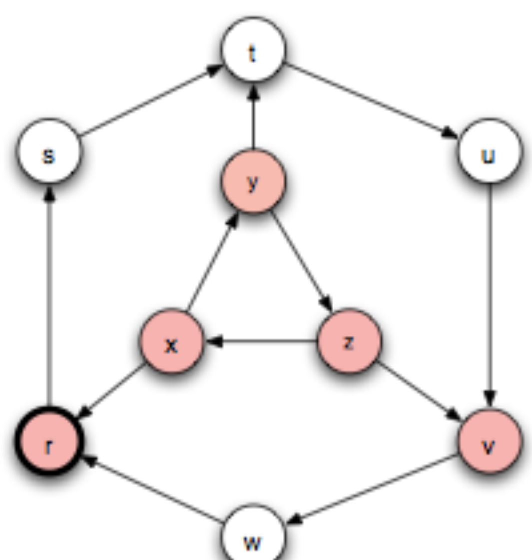
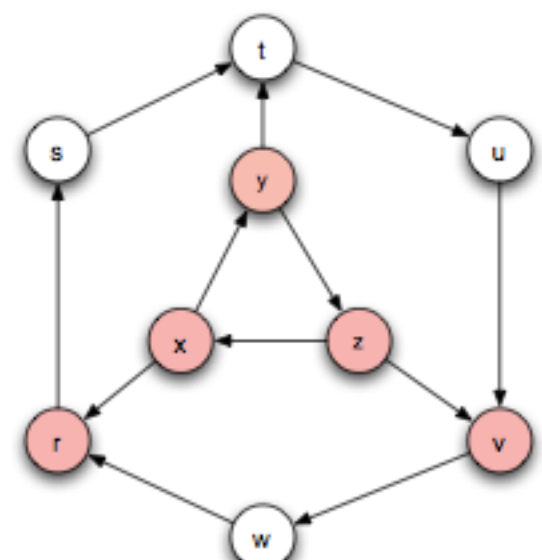


(c)



From the book *Networks, Crowds, and Markets: Reasoning about a Highly Connected World.* By David Easley and Jon Kleinberg. Cambridge University Press, 2010. Complete preprint on-line at http://www.cs.cornell.edu/home/kleinber/networks-book/

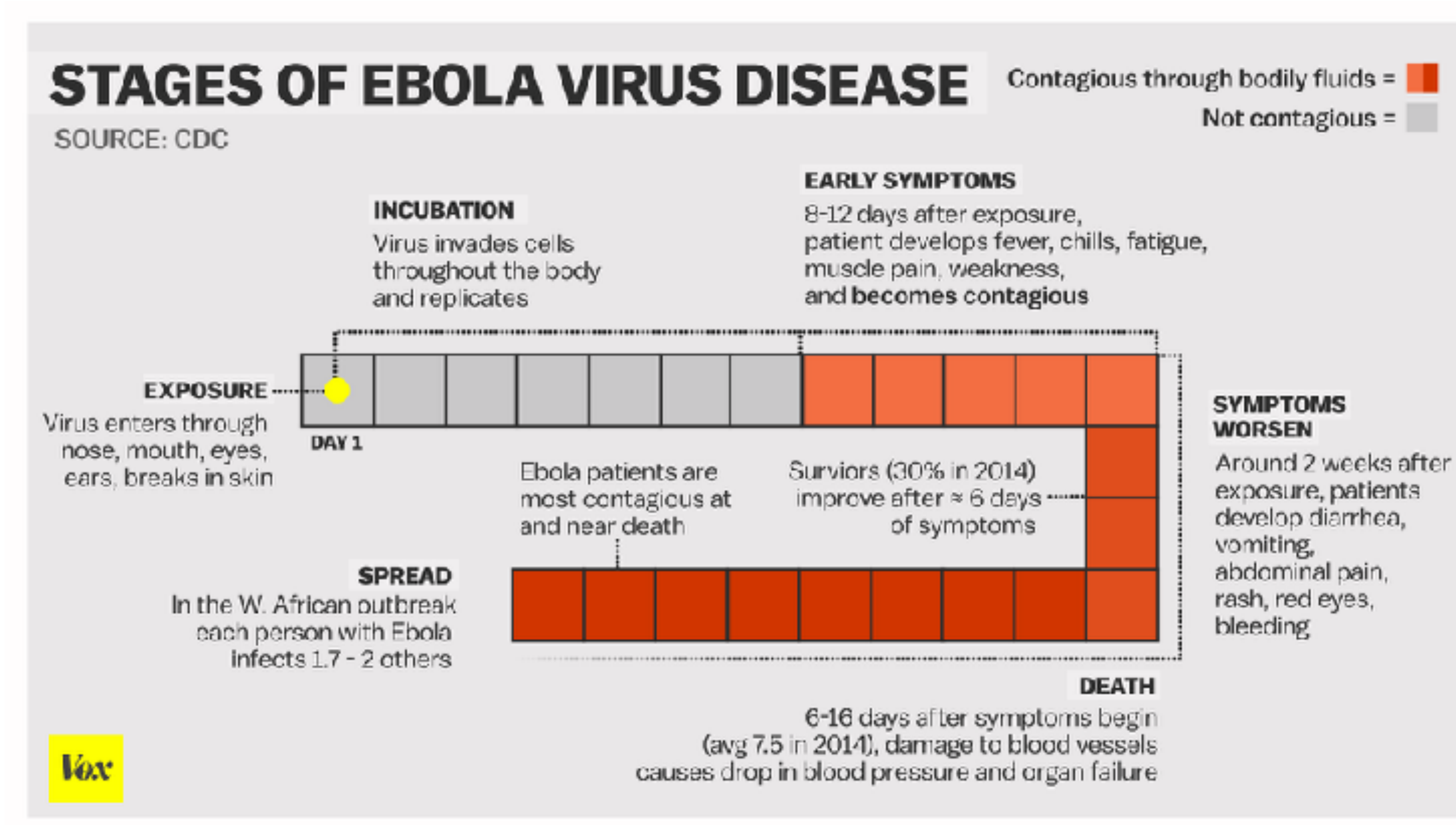
Individual-level network models of disease transmission

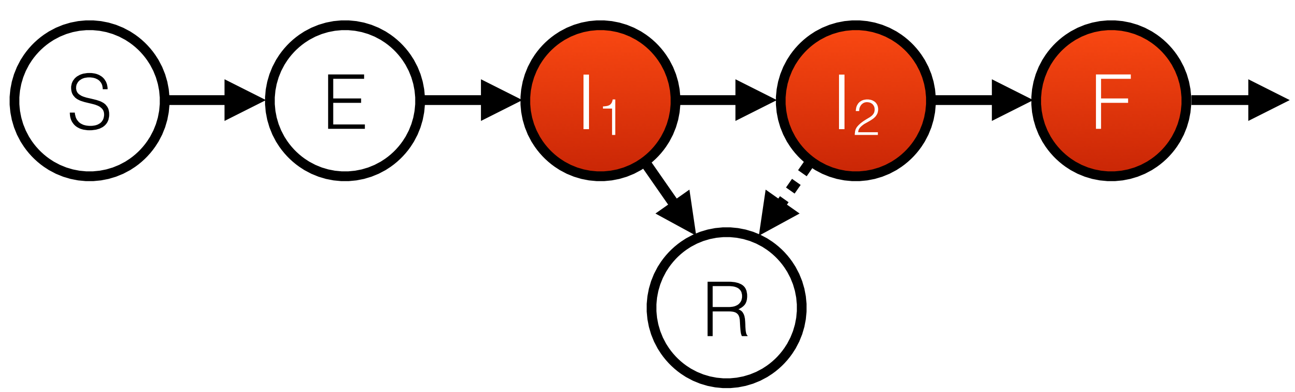
- Virus on a network example in NetLogo models library
- PyCX has several examples
- Let’s code one together!

Population level network model of disease transmission

- Can model population transmission on a network as an agent-based model or non-agent based model (e.g. ODE, stochastic model)

EVD in West Africa





D'Silva JP, Eisenberg MC. Modeling spatial invasion of Ebola in West Africa. Journal of theoretical biology. 2017 Sep 7;428:65-75.

Model Equations

$$\frac{dS}{dt} = -(\beta_I I_1 + \beta_2 I_2 + \beta_F F)S$$
$$\frac{dE}{dt} = (\beta_I I_1 + \beta_2 I_2 + \beta_F F)S - \alpha E$$
$$\frac{dI_1}{dt} = \alpha E - \gamma_1 I_1$$
$$\frac{dI_2}{dt} = \delta_1 \gamma_1 I_1 - \gamma_2 I_2$$
$$\frac{dF}{dt} = \delta_2 \gamma_2 I_2 - \gamma_F F$$
$$\frac{dR}{dt} = (1 - \delta_1)\gamma_1 I_1 + (1 - \delta_2)\gamma_2 I_2 - \gamma_R R$$

$$\boxed{\mathcal{R}_0 = \frac{\beta_1}{\gamma_1} + \frac{\beta_2 \delta_1}{\gamma_2} + \frac{\beta_F \delta_1 \delta_2}{\gamma_F}}$$

Measure: cumulative cases & deaths

Reporting Rate & Fraction of the Population at Risk

Fraction of individuals who have become infected x Population at risk x Reporting rate

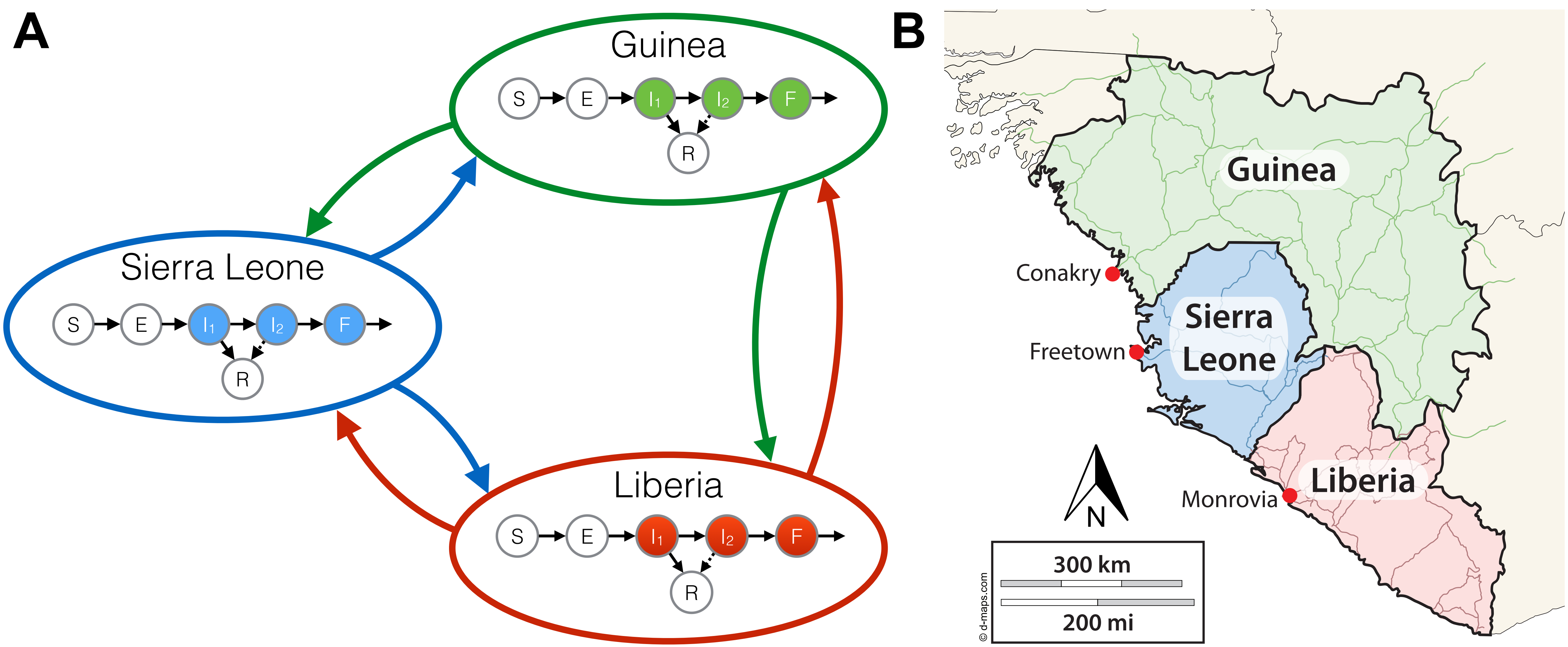
= Observed cases

Reporting Rate & Fraction of the Population at Risk

$$\overbrace{\text{Fraction of individuals who have become infected}}^{\text{Model}} \times \overbrace{\text{Population at risk} \times \text{Reporting rate}}^{\mathbf{k}}$$

$$= \text{Observed cases}$$

Spatial network



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Gravity Model

- Model of transmission or movement between locations
- Suppose that contact is higher with regions that are larger (population centers), and regions that are closer
- Scale transmission or movement using ‘gravity’ term:

$$\theta_{ij} = \frac{N_i N_j}{d_{ij}^2}$$

$$\dot{S}_n = -(\lambda_n + \lambda_m + \lambda_l)S_n$$
$$\dot{E}_n = (\lambda_n + \lambda_m + \lambda_l)S_n - \alpha E_n$$
$$\dot{I}_{1n} = \alpha E_n - \gamma_n I_{1n} - r_{1,n} I_{1n}$$
$$\dot{I}_{2n} = \gamma_n I_{1n} - \delta I_{2n} - r_{2,n} I_{2n}$$
$$\dot{F}_n = \delta I_{2n} - \delta_2 F_n$$
$$\dot{R}_n = r_{1,n} I_{1n} + r_{2,n} I_{2n}$$
$$\dot{IC}_n = k_{norm} \alpha E_n$$
$$\dot{DC}_n = k_{norm} \delta I_{2n}$$
$$\lambda_n = \beta_{1,n} I_{1n} + \beta_{2,n} I_{2n} + \beta_{F,n} F_n$$
$$\lambda_m = \theta_{n,m}(\beta_{1,n} I_{1m} + \beta_{2,n} I_{2m} + \beta_{F,n} F_m)$$
$$\lambda_l = \theta_{n,l}(\beta_{1,n} I_{1l} + \beta_{2,n} I_{2l} + \beta_{F,n} F_l)$$

$$\theta_{n,m} = \kappa_n \frac{\rho_n \rho_m}{(d_{n,m})^{\iota}}$$

$$\theta_{n,l} = \kappa_n \frac{\rho_n \rho_l}{(d_{n,l})^{\iota}}$$

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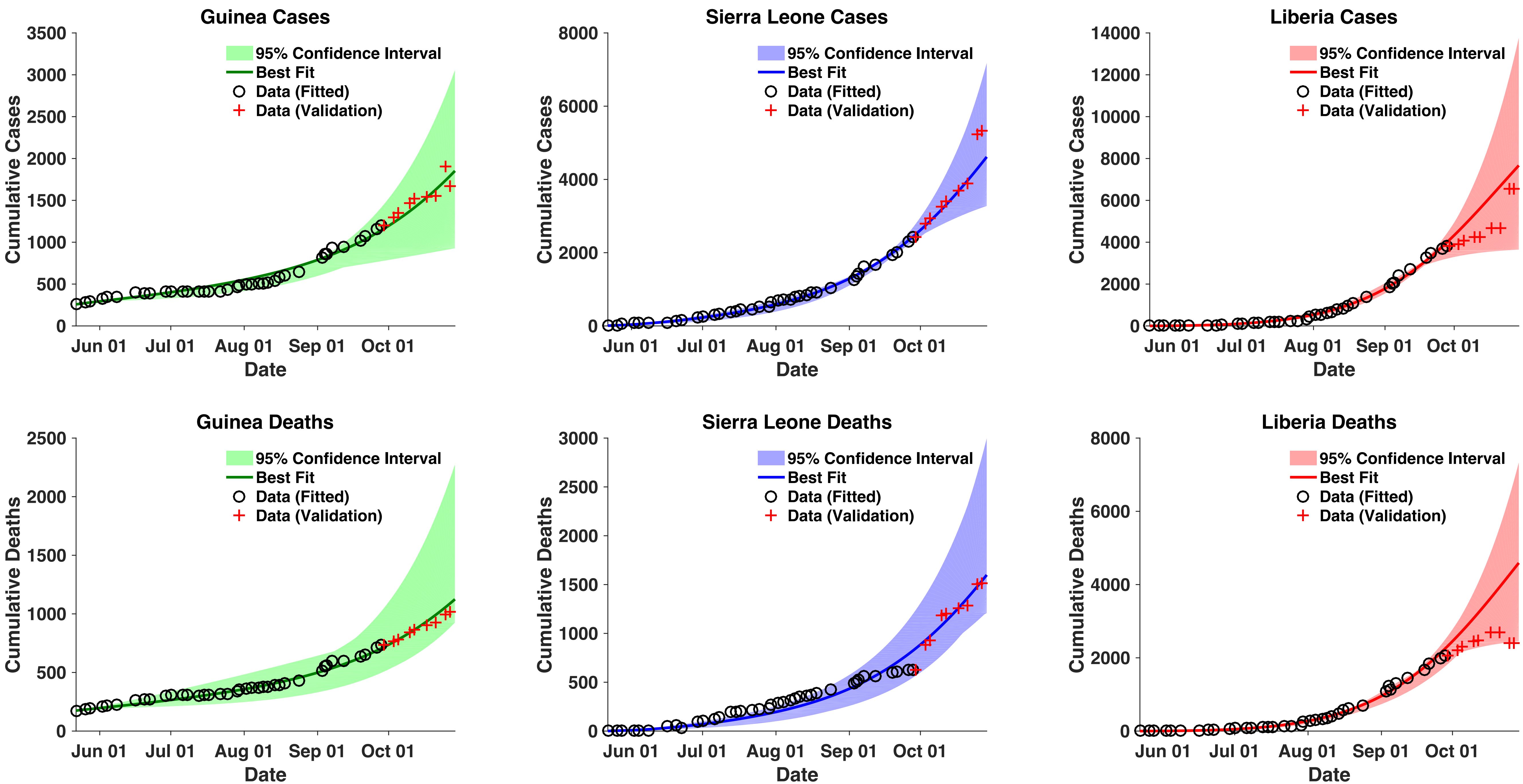
Parameter Estimation

- Estimate parameters from incidence data on cases and deaths
- Some parameter information from the literature and from ongoing reporting of incubation period, infectious period, etc.
- Extensive uncertainty and issues of unidentifiability!
    - Many different parameter values will fit the data equally (or close to equally) well



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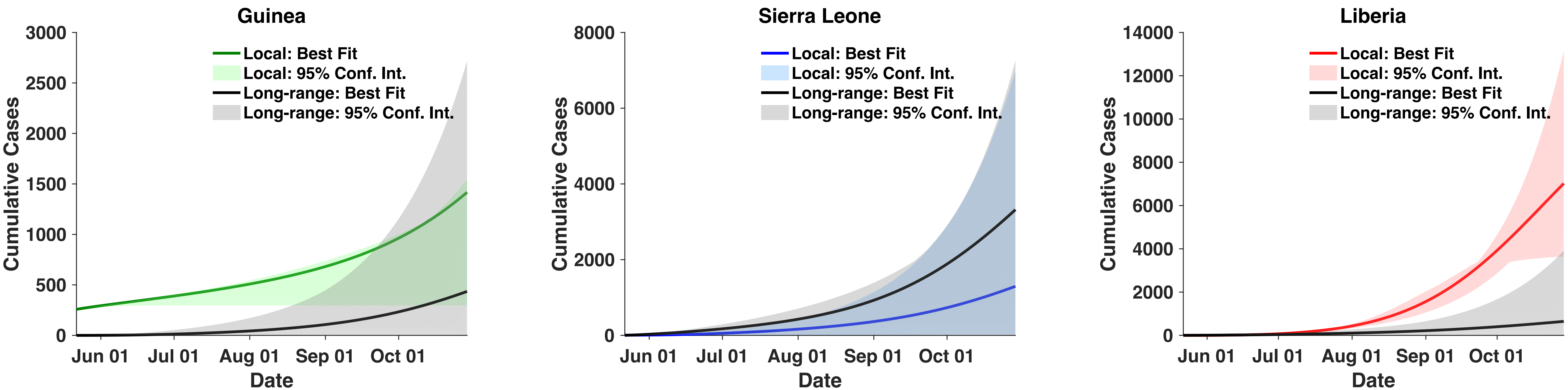

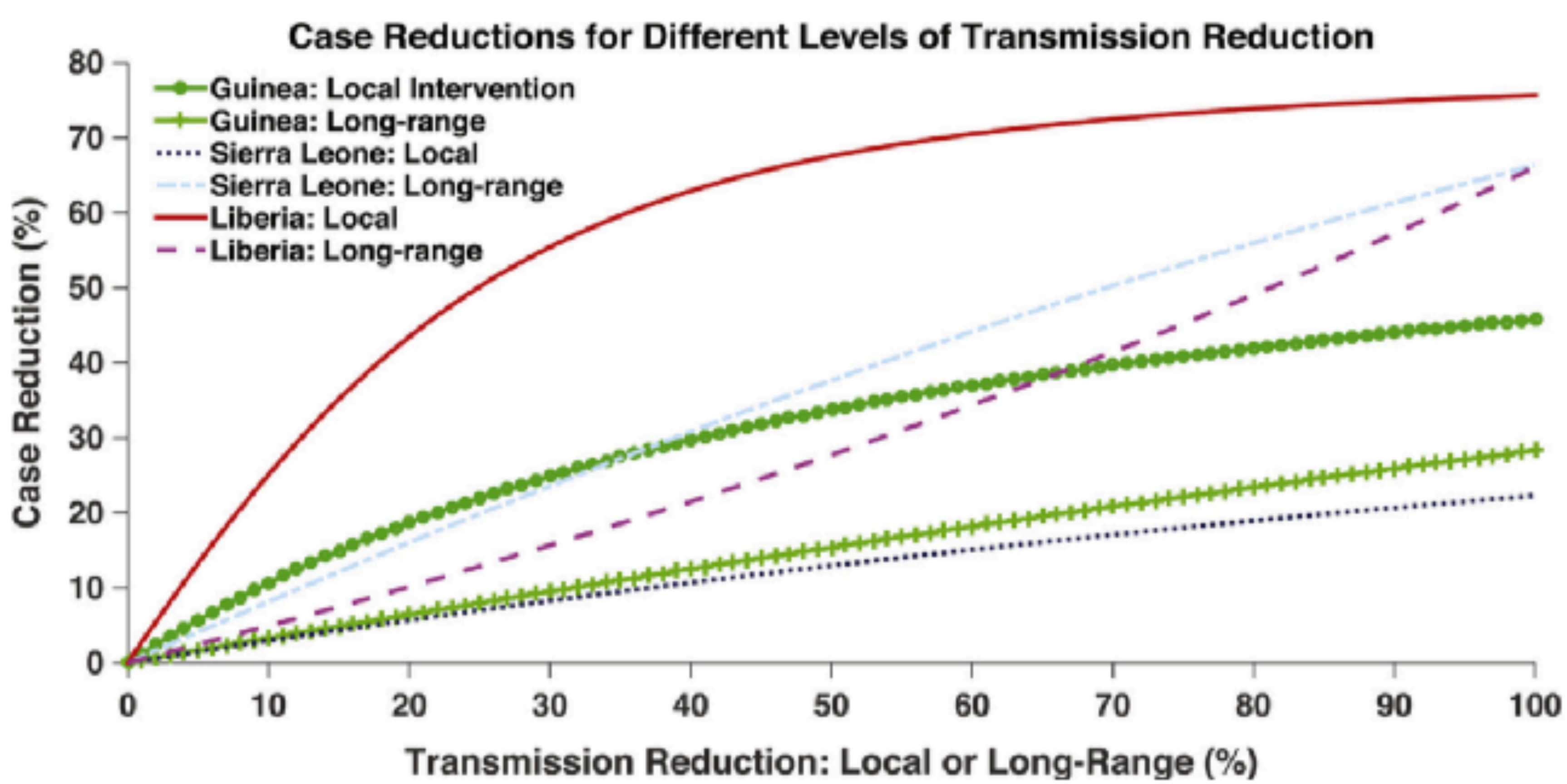

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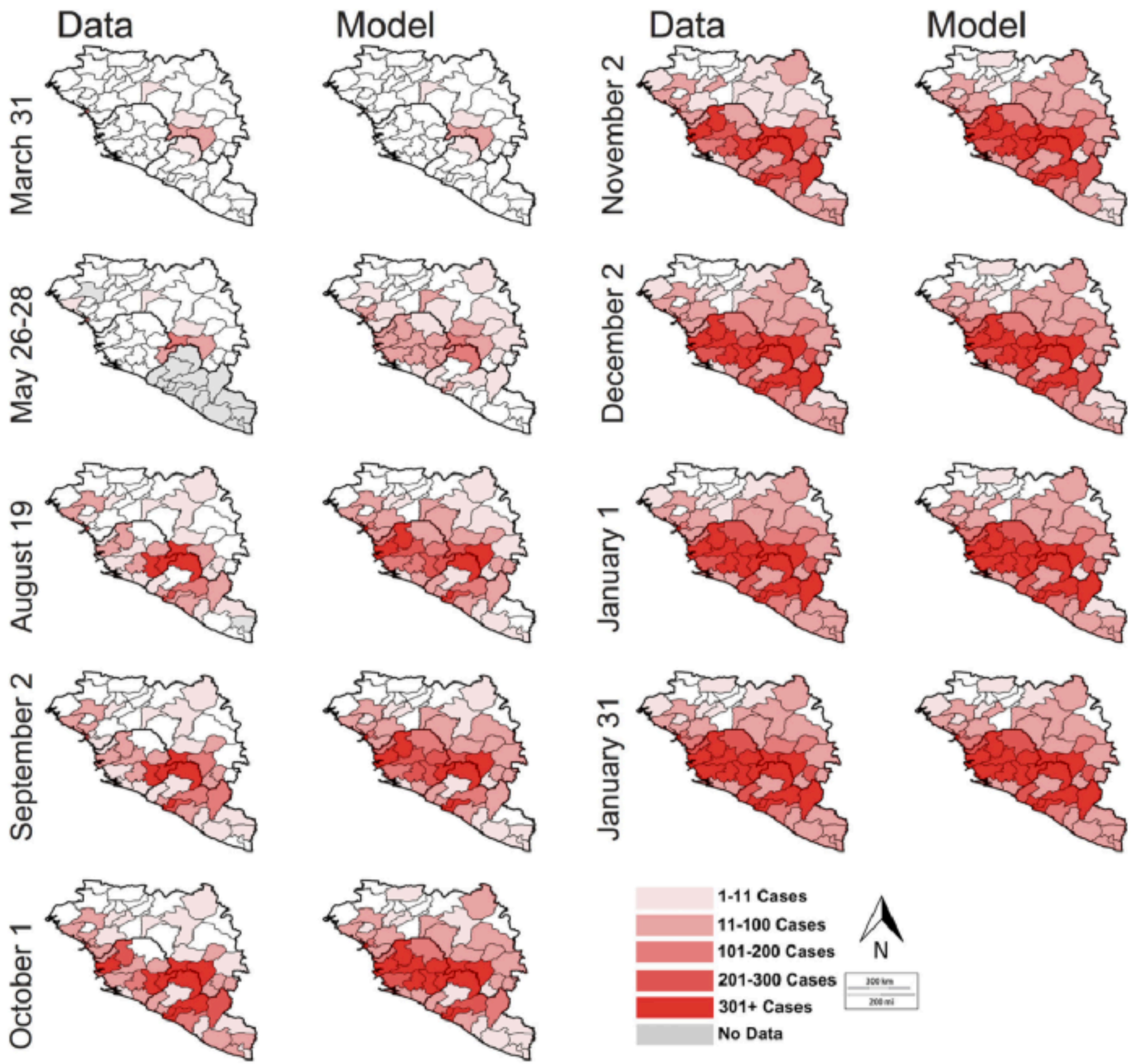
More granular: modeling at the district level

- Extend the model to the 63 districts in Guinea, Liberia, and Sierra Leone

- Adapt the model to be stochastic (since some districts have small population)



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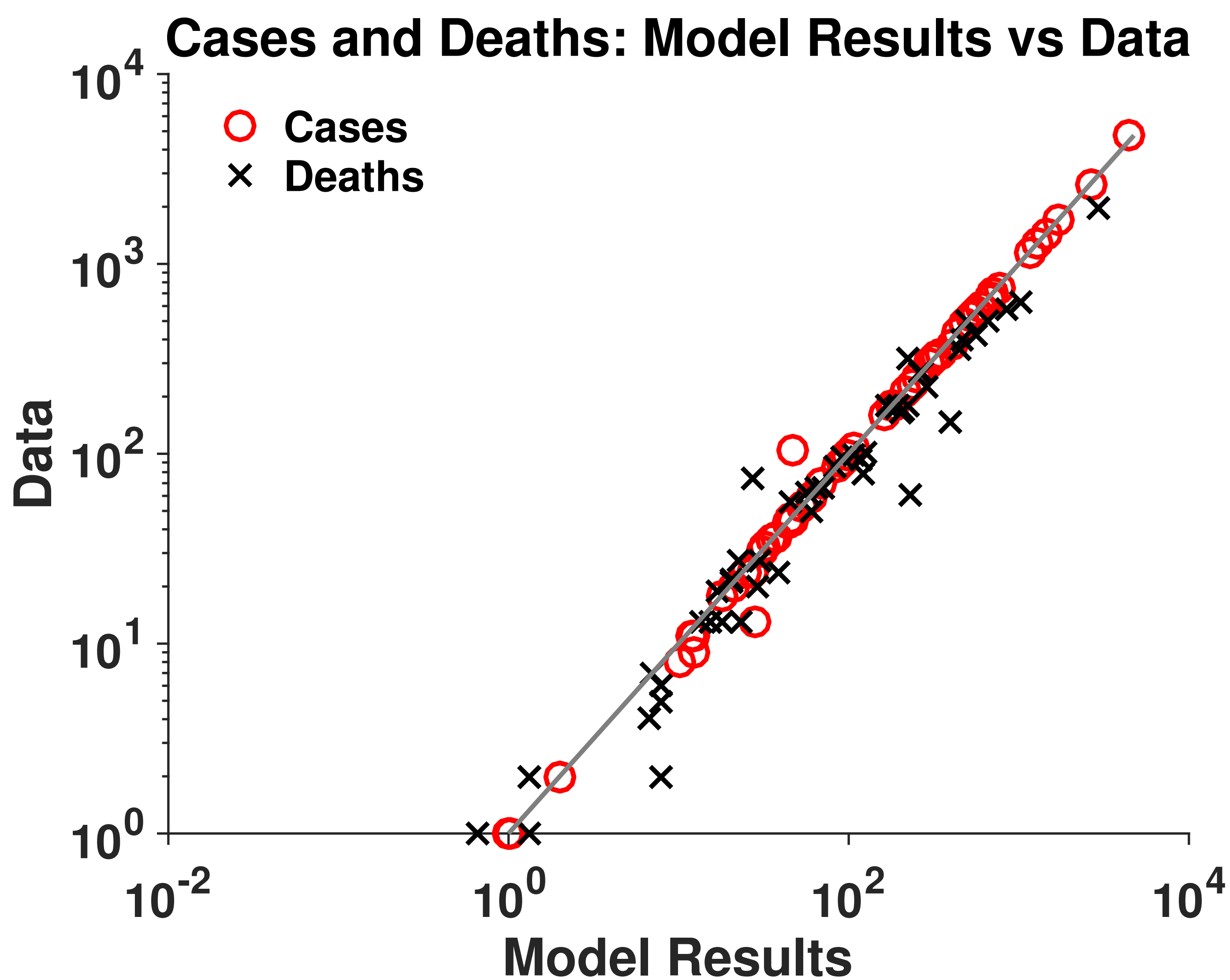

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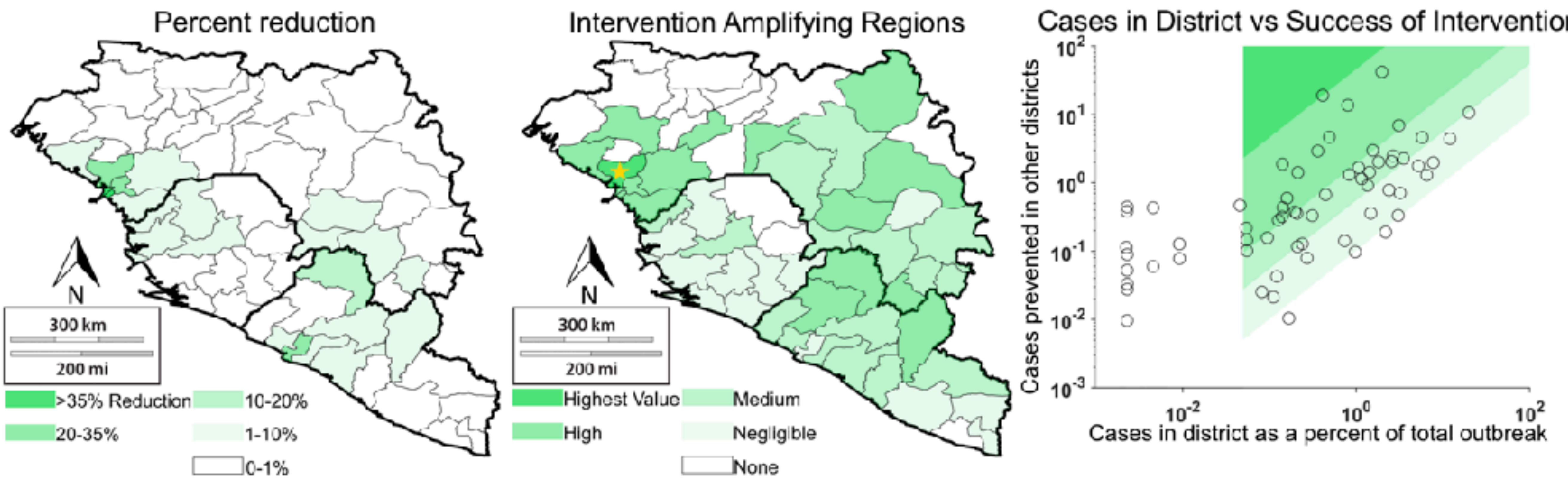

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Epidemic Dynamics on Networks

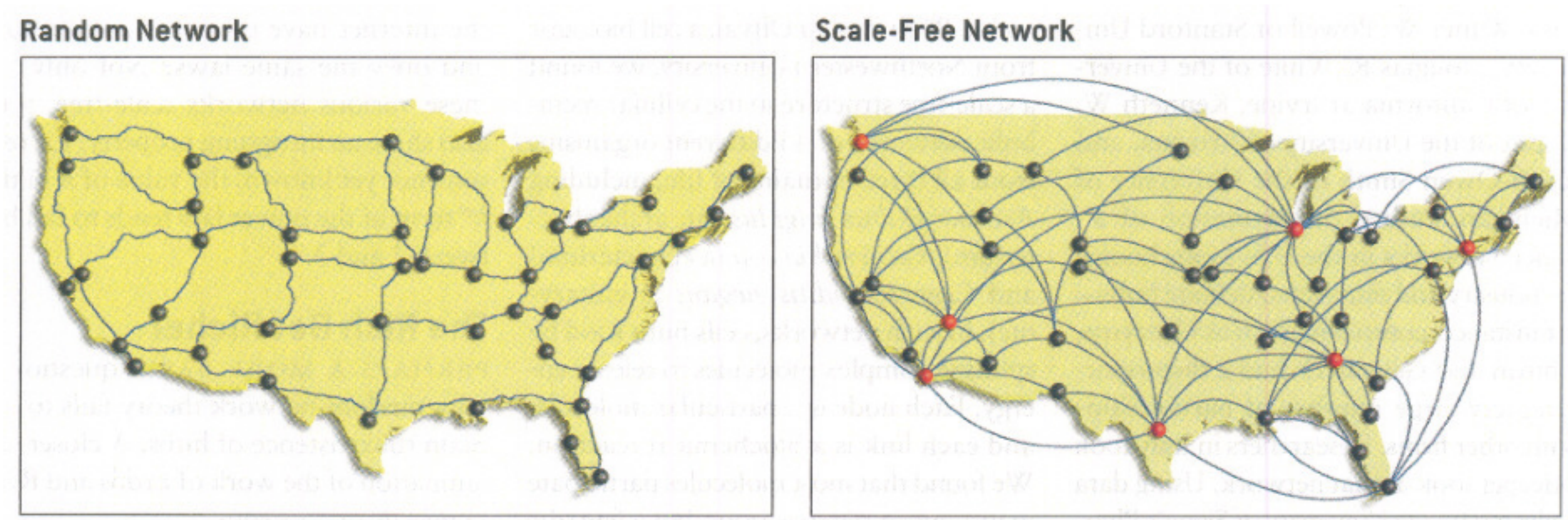
- Network structure plays a huge role on the epidemic dynamics
    - Hubs, sparsely connected, etc.
- Small world property can tend to produce synchronized epidemics (e.g. oscillations)

Epidemic Dynamics on Networks

- Where you place high-risk individuals or patches can significantly affect R_0, disease dynamics, etc.
    - E.g. if cluster high-risk nodes together vs spread apart
    - If hub vs periphery is infected - the scale free vulnerability to hub attacks

Epidemic Dynamics on Networks

- How would interventions/risk/dynamics differ for epidemic spread by roads vs air travel? (and what does this mean for pandemics & emerging diseases/behaviors)



Epidemic Dynamics on Networks

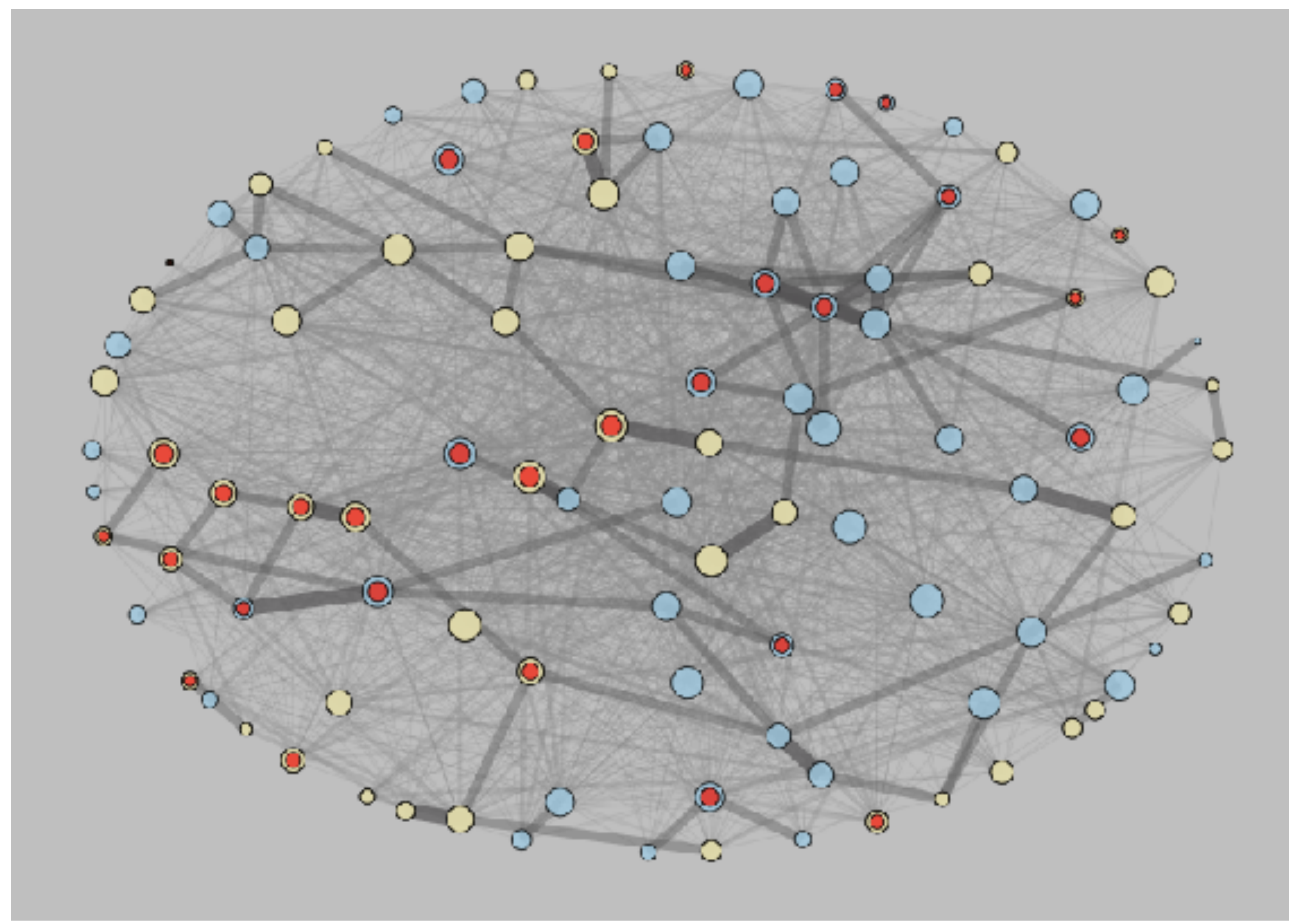
- Major & still very open area of research
- Can have significant impact on interventions & control strategies
    - Should you target well-connected individuals?
    - Are there specific network structures you should look for as high-risk?

Epidemic Dynamics on Networks

- Lots of interesting data to work with too—can often track contacts, etc.

- Example: the eX-FLU study (Aiello et al.)

- Substudy tracking contacts using Bluetooth from cell phones



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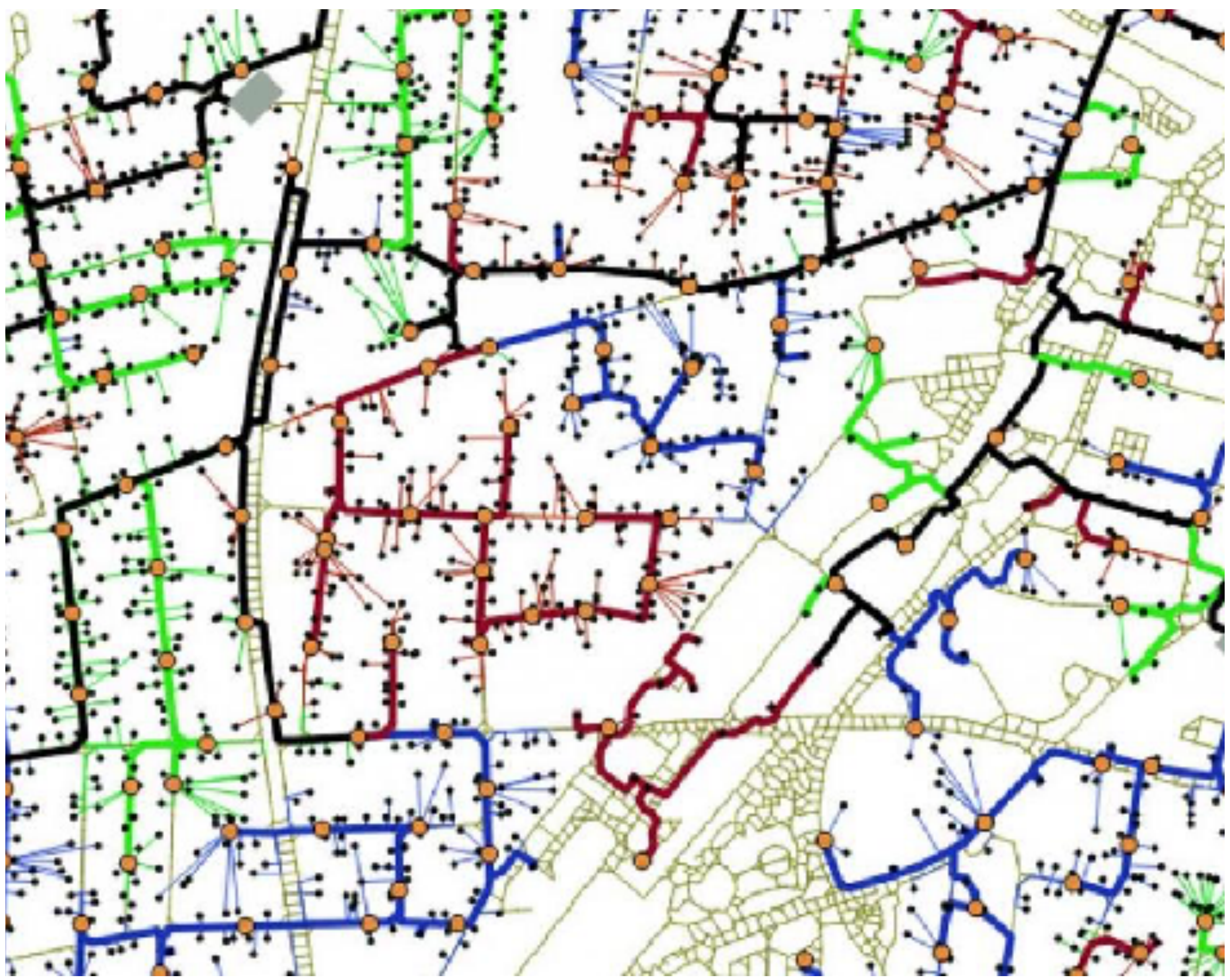
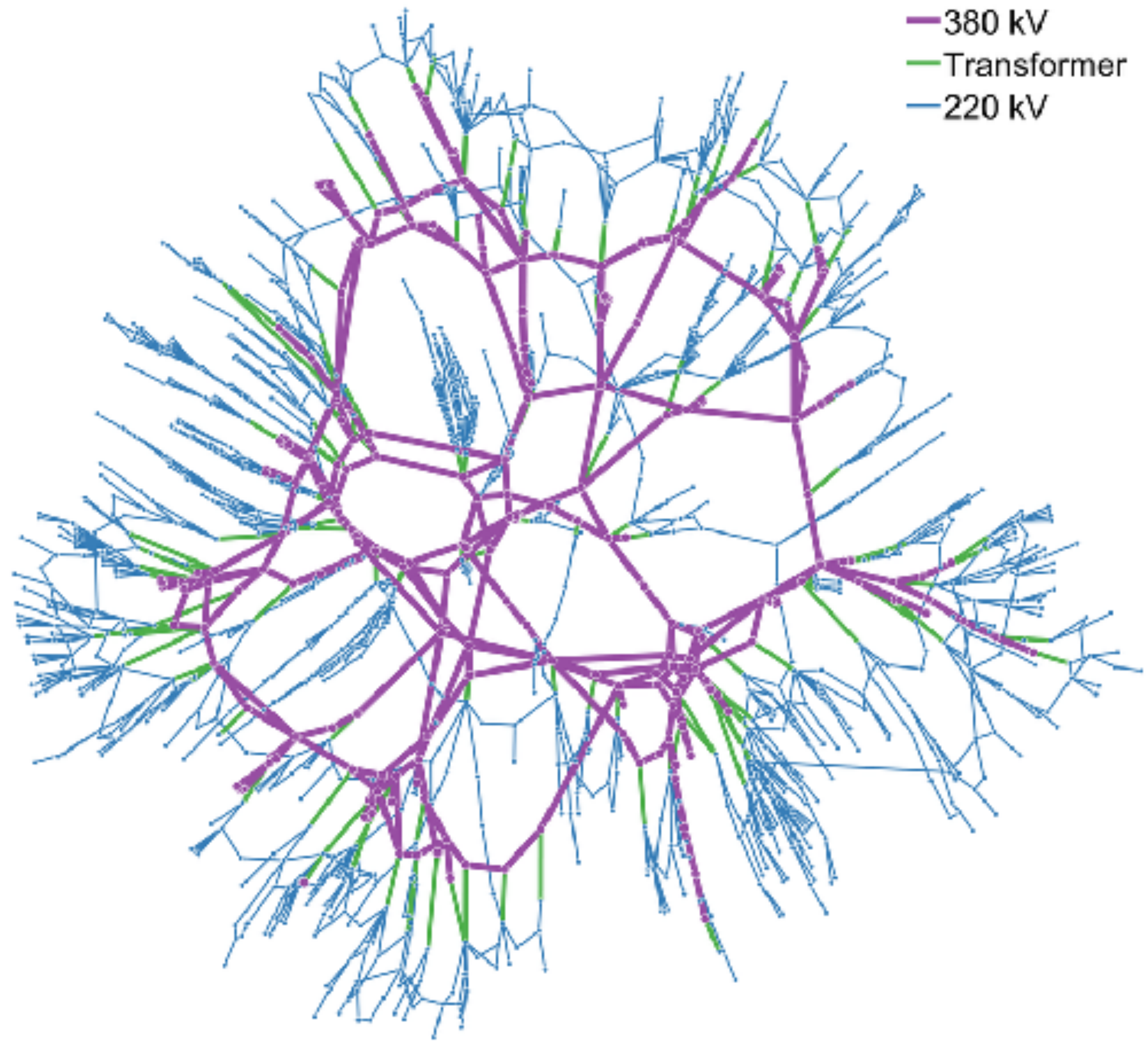
Example: power grids





SMART-DS: Synthetic Models for Advanced, Realistic Testing: Distribution Systems and Scenarios
https://www.nrel.gov/grid/smart-ds.html

By Paul Cuffe - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=70226122

Example: Neuronal networks

- Firing dynamics on networks used extensively in mathematical/ computational neuroscience

- Example: **ring model of direction sense!**

- Proposed as a model in the 1990's

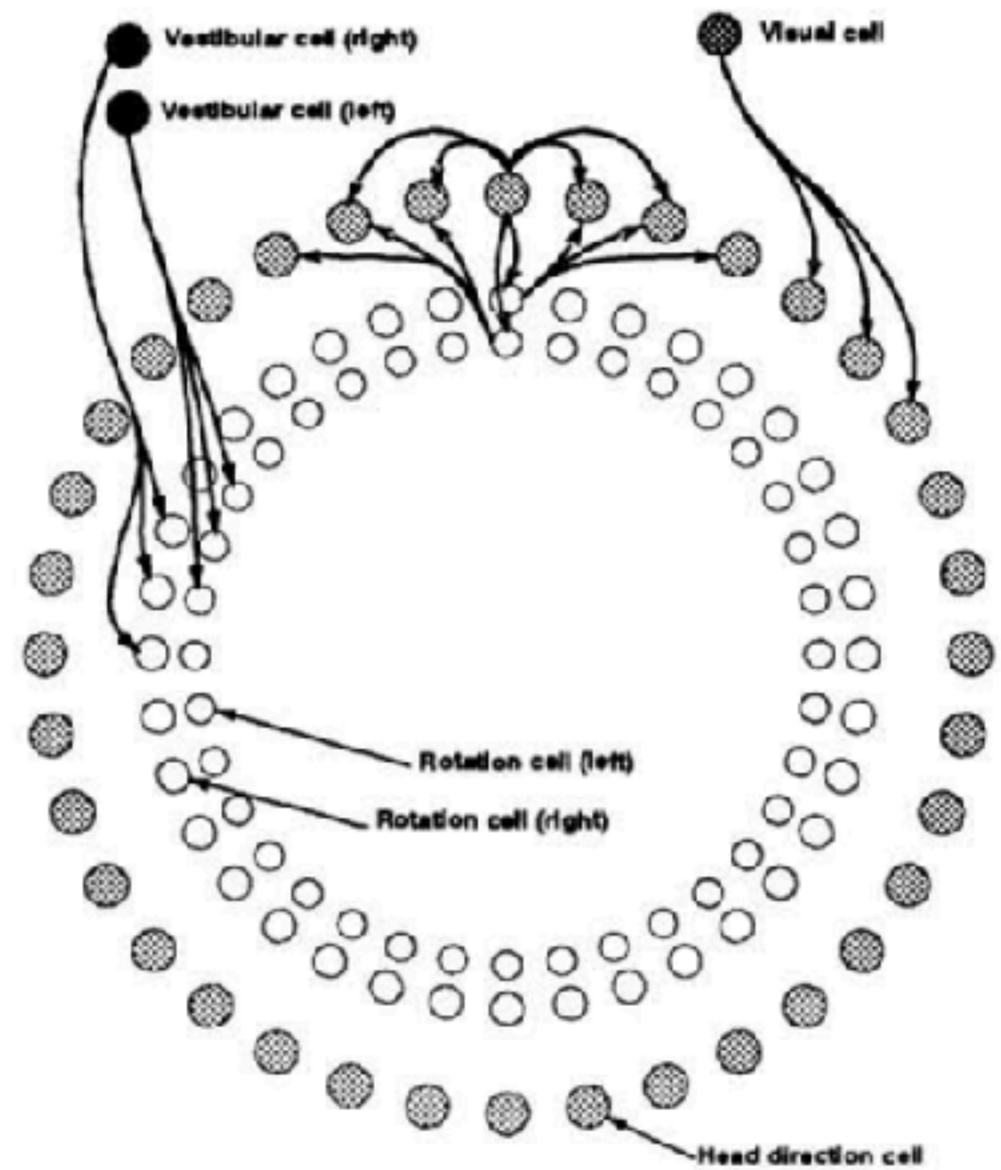
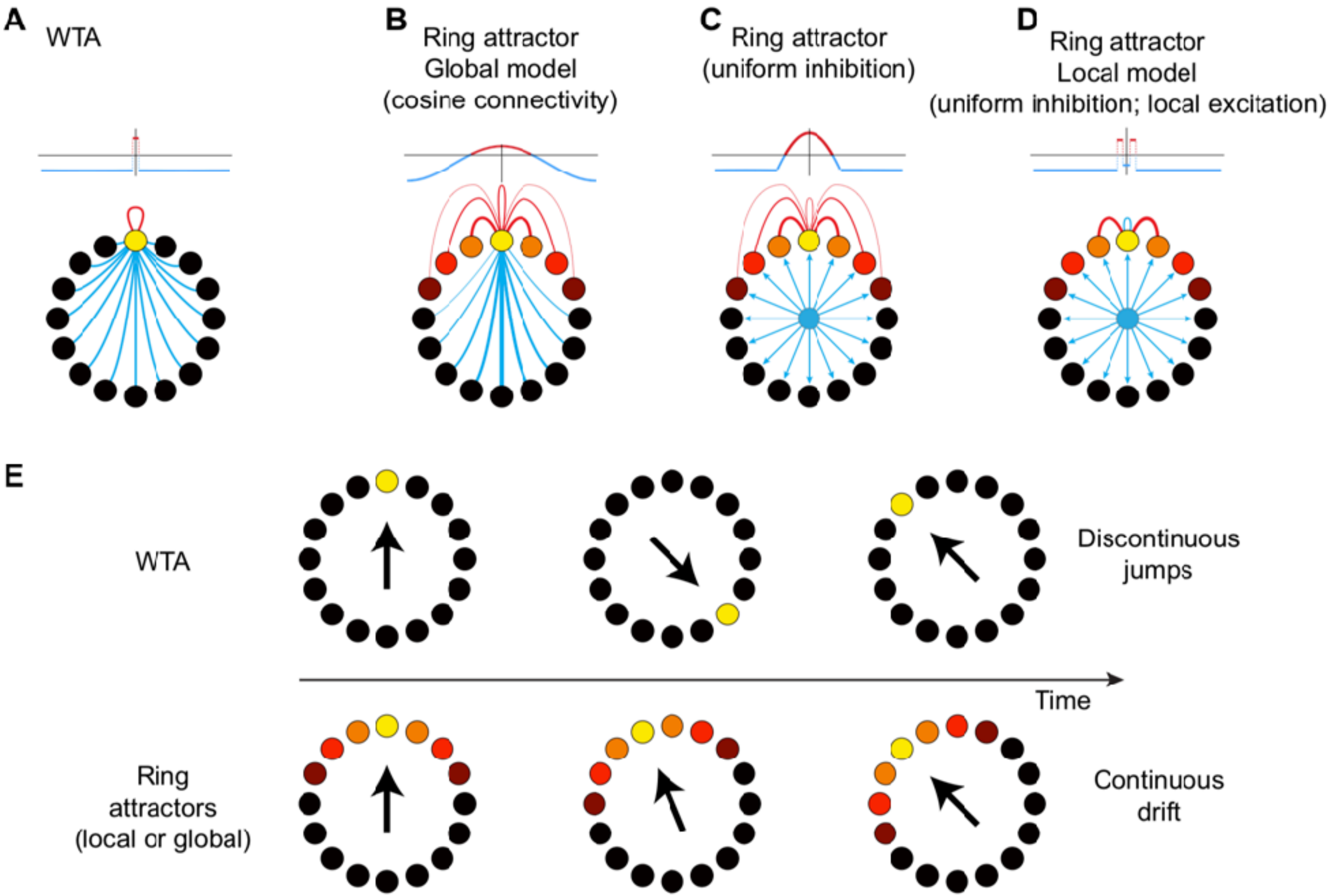


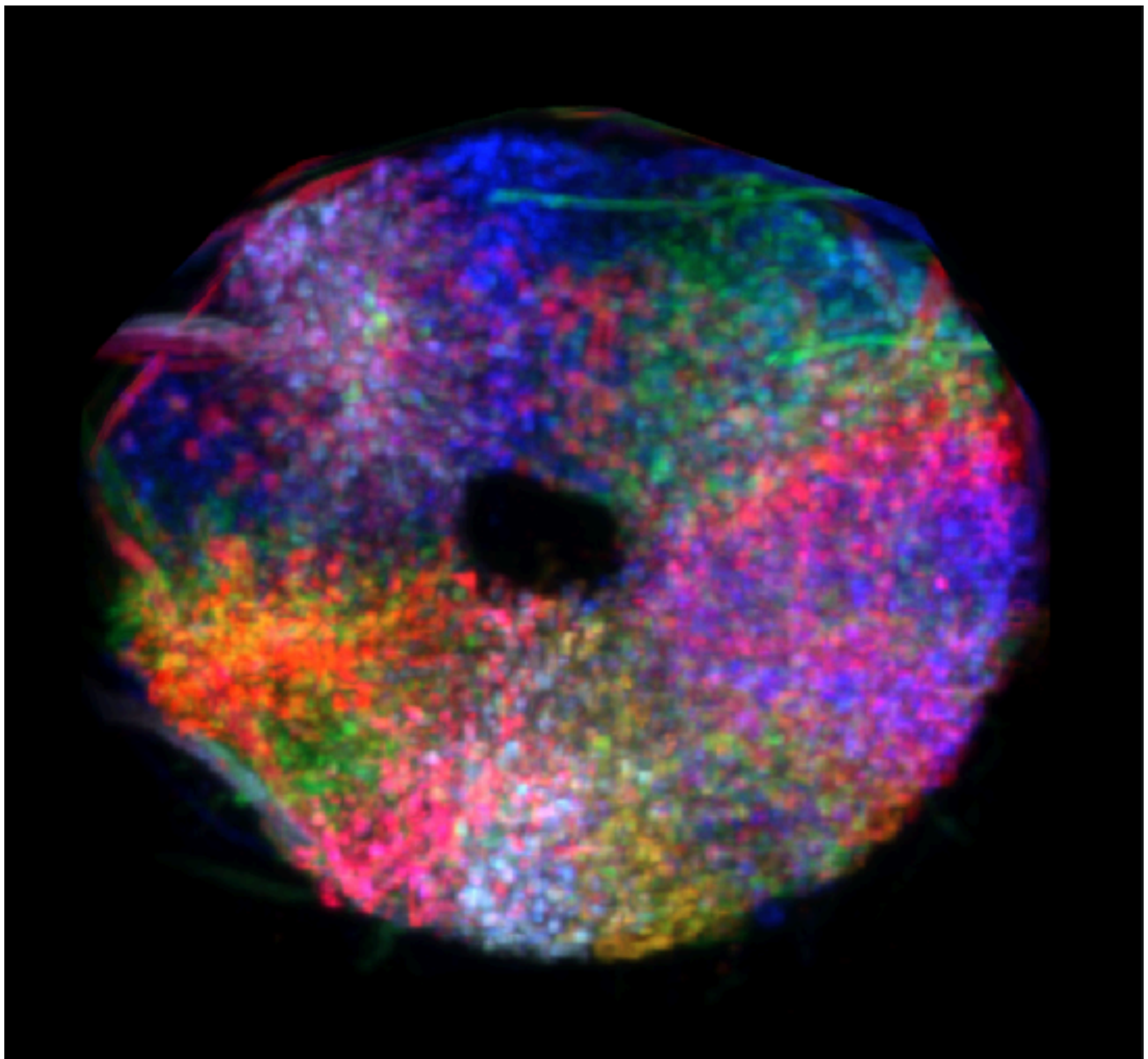
Figure 3: Architecture of the head direction cell model.

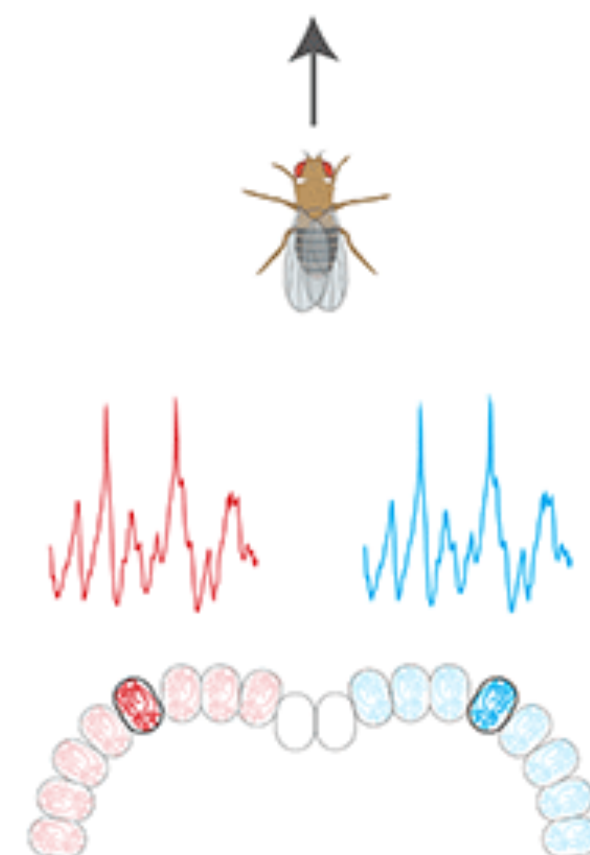
Skaggs, WE., et al. "A model of the neural basis of the rat's sense of direction." *Advances in neural information processing systems*. 1995.



Kim, Sung Soo, et al. "Ring attractor dynamics in the Drosophila central brain." Science 356.6340 (2017): 849-853.

2017: Found the ring network in *Drosophila* (fruit fly)!





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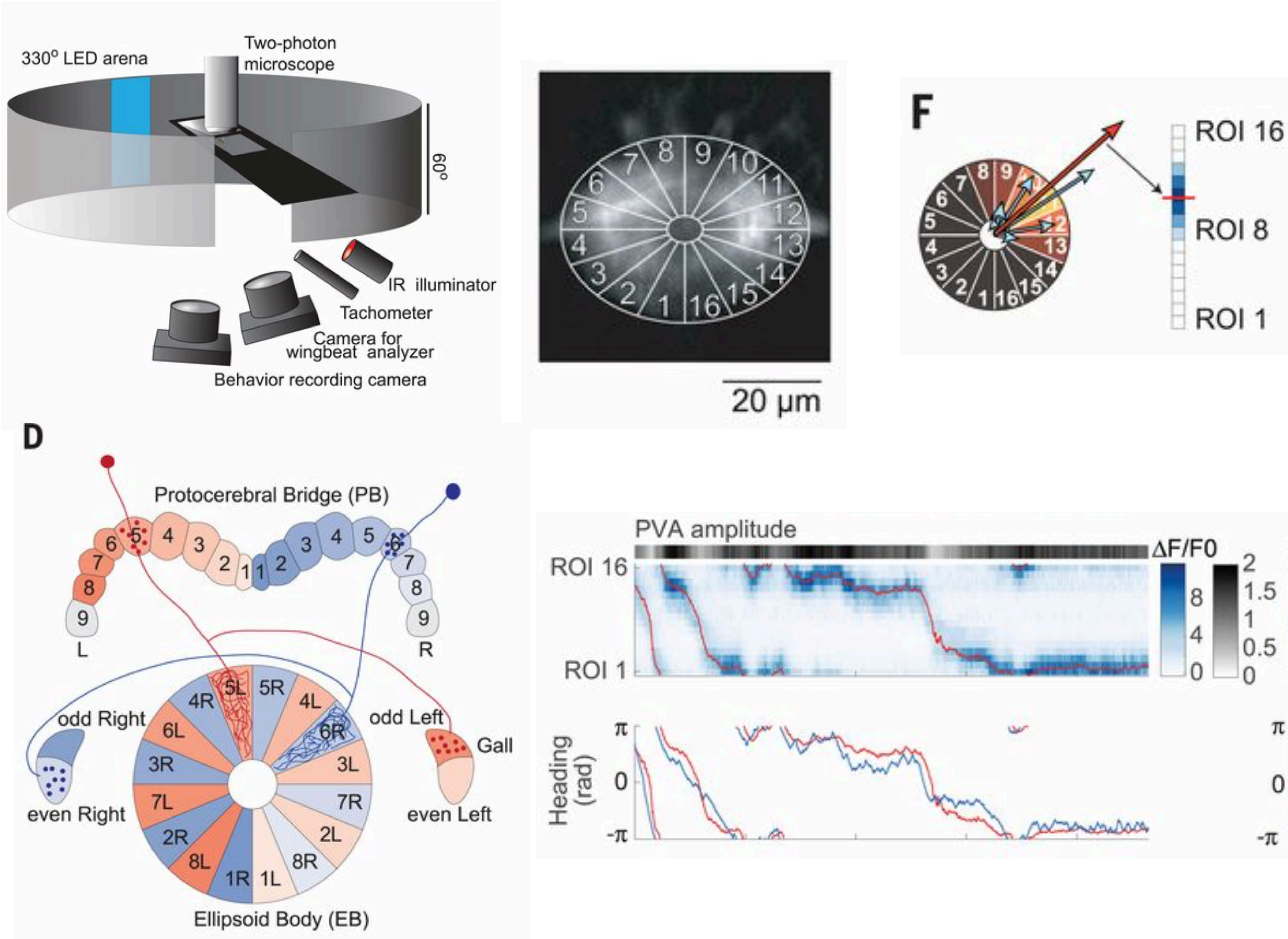
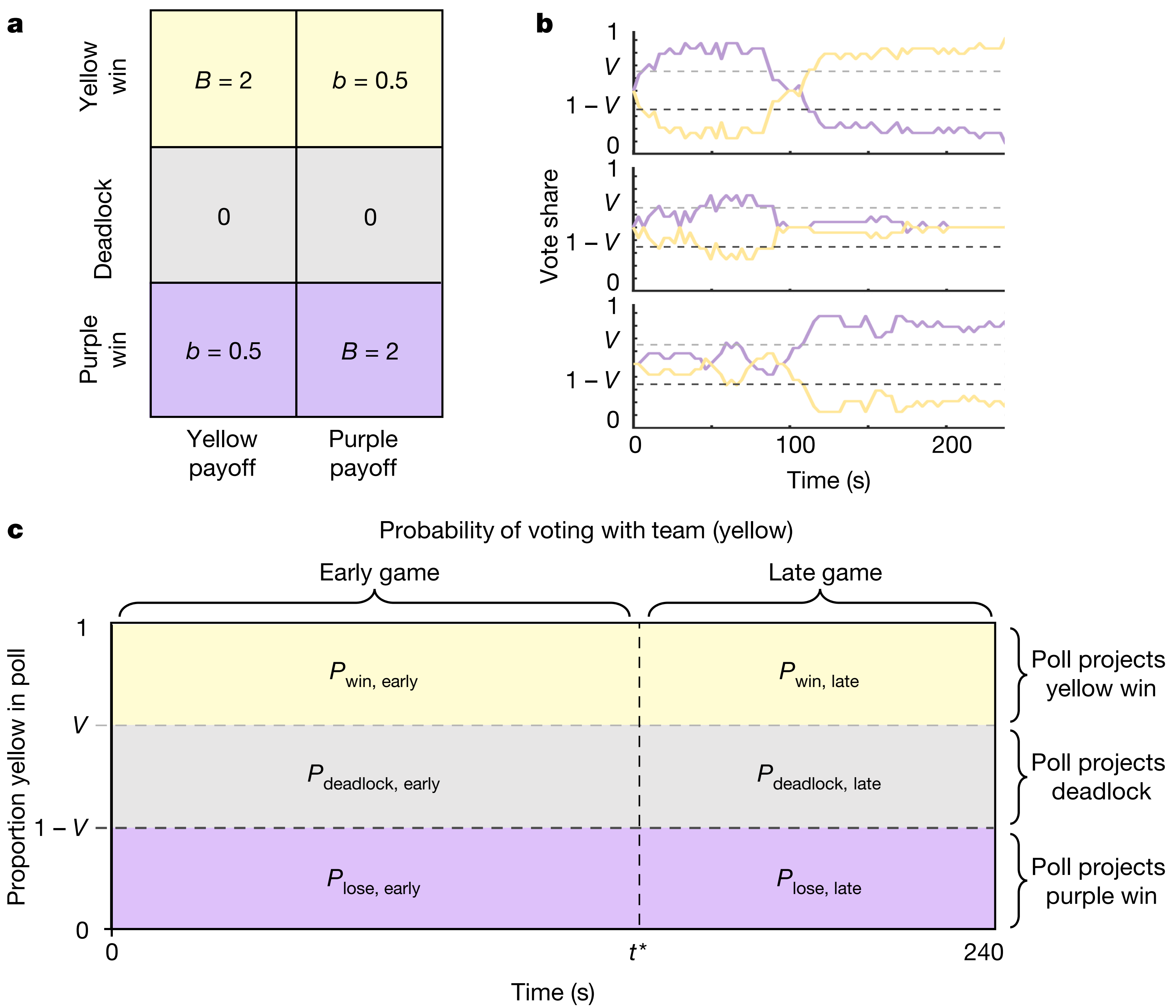


330° LED arena
Two-photon microscope
60°
IR illuminator
Tachometer
Camera for wingbeat analyzer
Behavior recording camera
8 9 10 7 11 6 12 5 13 4 14 3 15 2 1 16
20 μm
F
ROI 16
ROI 8
ROI 1
D
Protocerebral Bridge (PB)
9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9
L
R
5L 5R 4R 4L 6L 6R 3R 3L 7L 7R 2R 2L 8L 8R 1R 1L
odd Right
odd Left
Gall
even Right
even Left
Ellipsoid Body (EB)
PVA amplitude
ΔF/F0
ROI 16
ROI 1
8 4 0
2 1.5 1 0.5 0
Heading (rad)
π
0
-π

Example: information gerrymandering



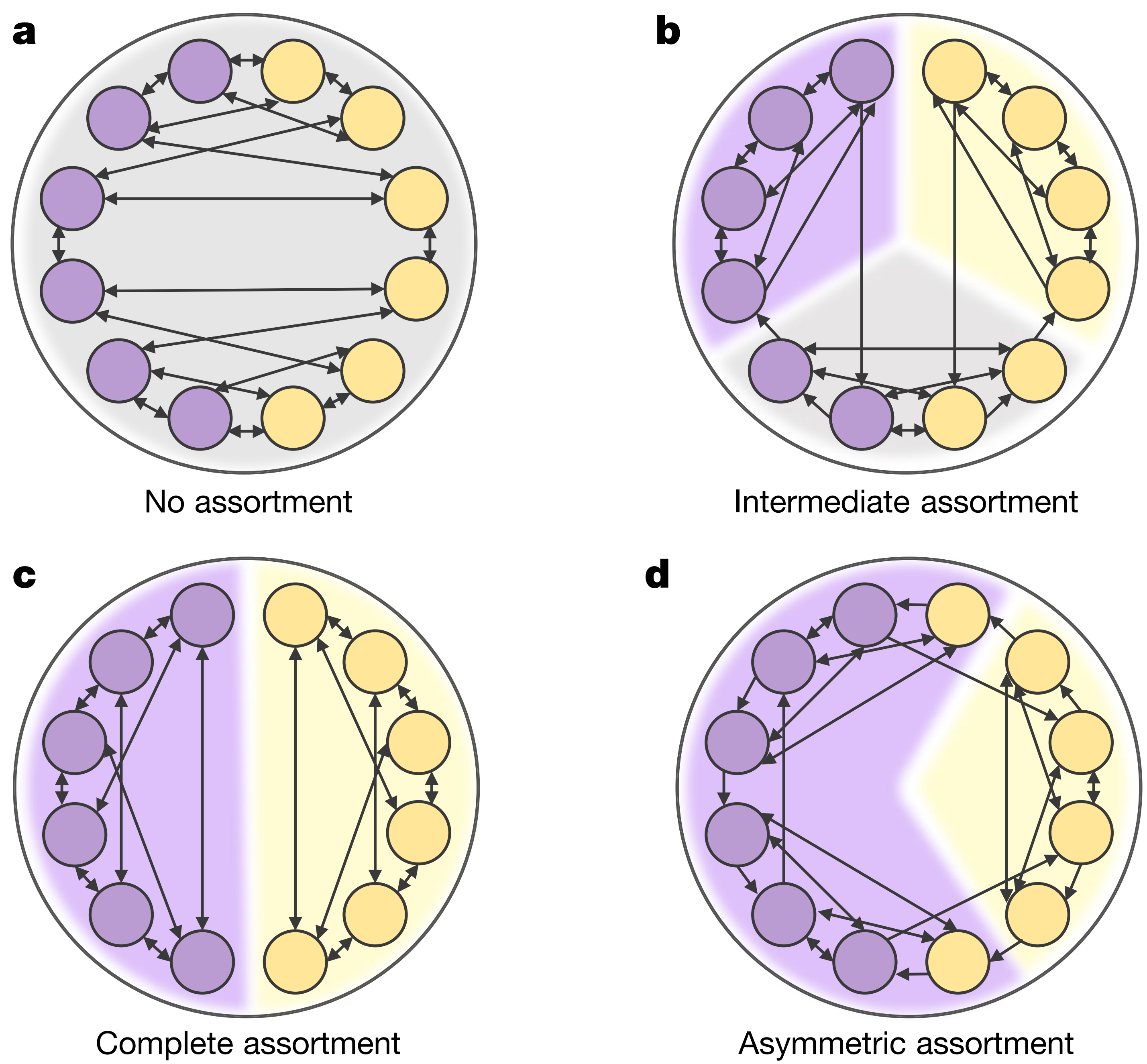
Stewart, Alexander J., et al. "Information gerrymandering and undemocratic decisions." Nature 573.7772 (2019): 117-121.

Electoral Gerrymandering

Consider 24 people, 12 favoring the Purple party
and 12 favoring the Yellow party

Stewart, Alexander J., et al. "Information gerrymandering and undemocratic decisions." Nature 573.7772 (2019): 117-121.

Network influence assortment



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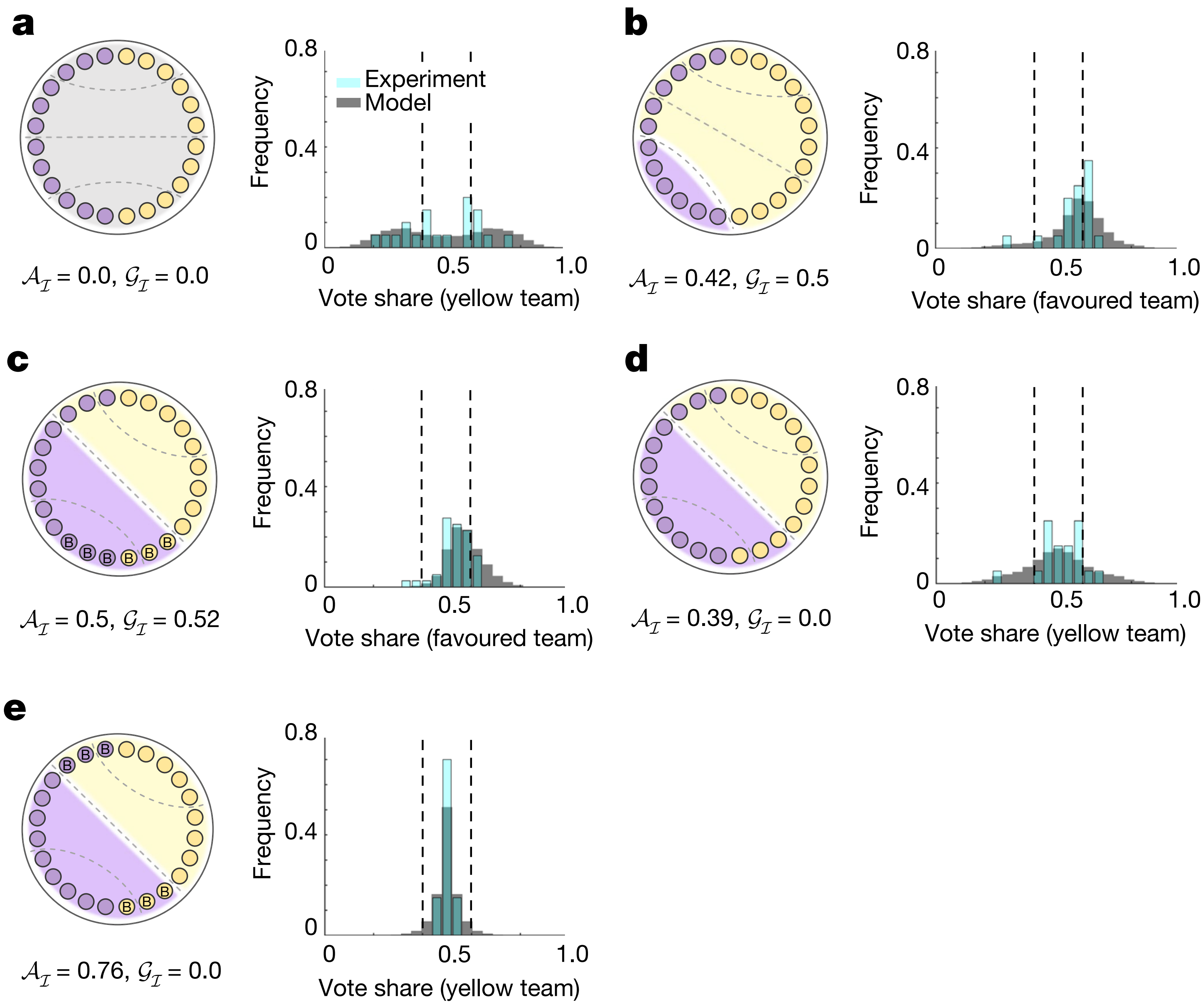
Experimental data



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Examples

- Percolation on a network
- Diffusion on a network (movement, etc.)
- Regulatory relationships in cells (levels of gene activity, protein concentrations, etc.)
- Ecological relationships (species populations)
- Coupled oscillators (e.g. fireflies etc)
  - https://ncase.me/fireflies/
  - PyCX example code

Dynamics **of** networks

- Things to consider
  - How do we add/remove nodes?
  - How do we add/remove edges?
- Dynamics of networks can often be framed as dynamics on networks where we activate/inactivate nodes/edges in a super-network
  - E.g. sexual network partnerships

Dynamics **of** networks

- Often depends on the question at hand—often the rules for changing network structure are often question and system specific

- Random graph generators from last time can also be thought of as dynamics of networks

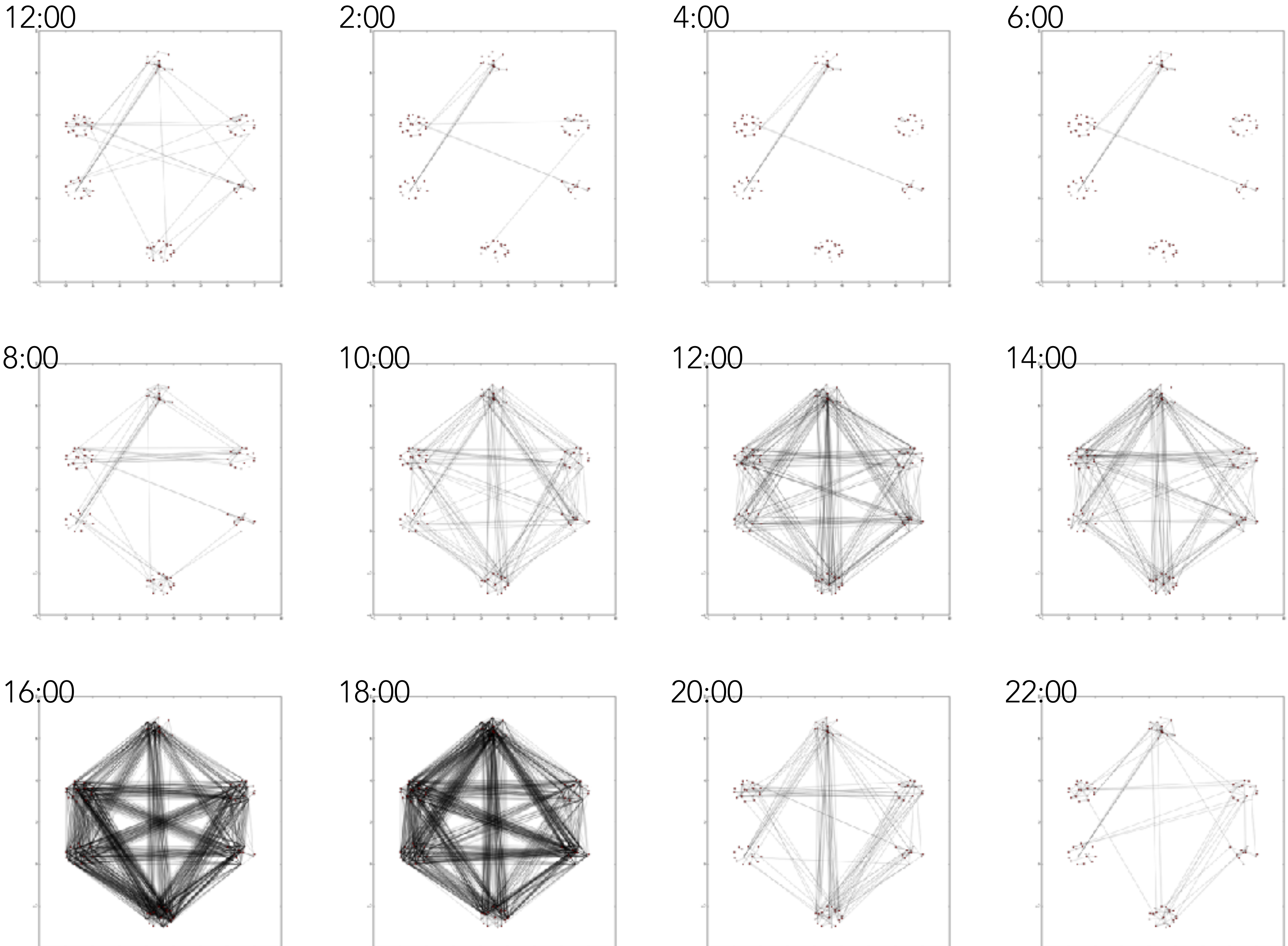
    - Erdös-Renyi

    - Small world

    - Preferential attachment

Dynamics **of** networks

- Dynamic empirical networks - contact networks, travel networks, ecological networks, trade networks, social media networks, etc.



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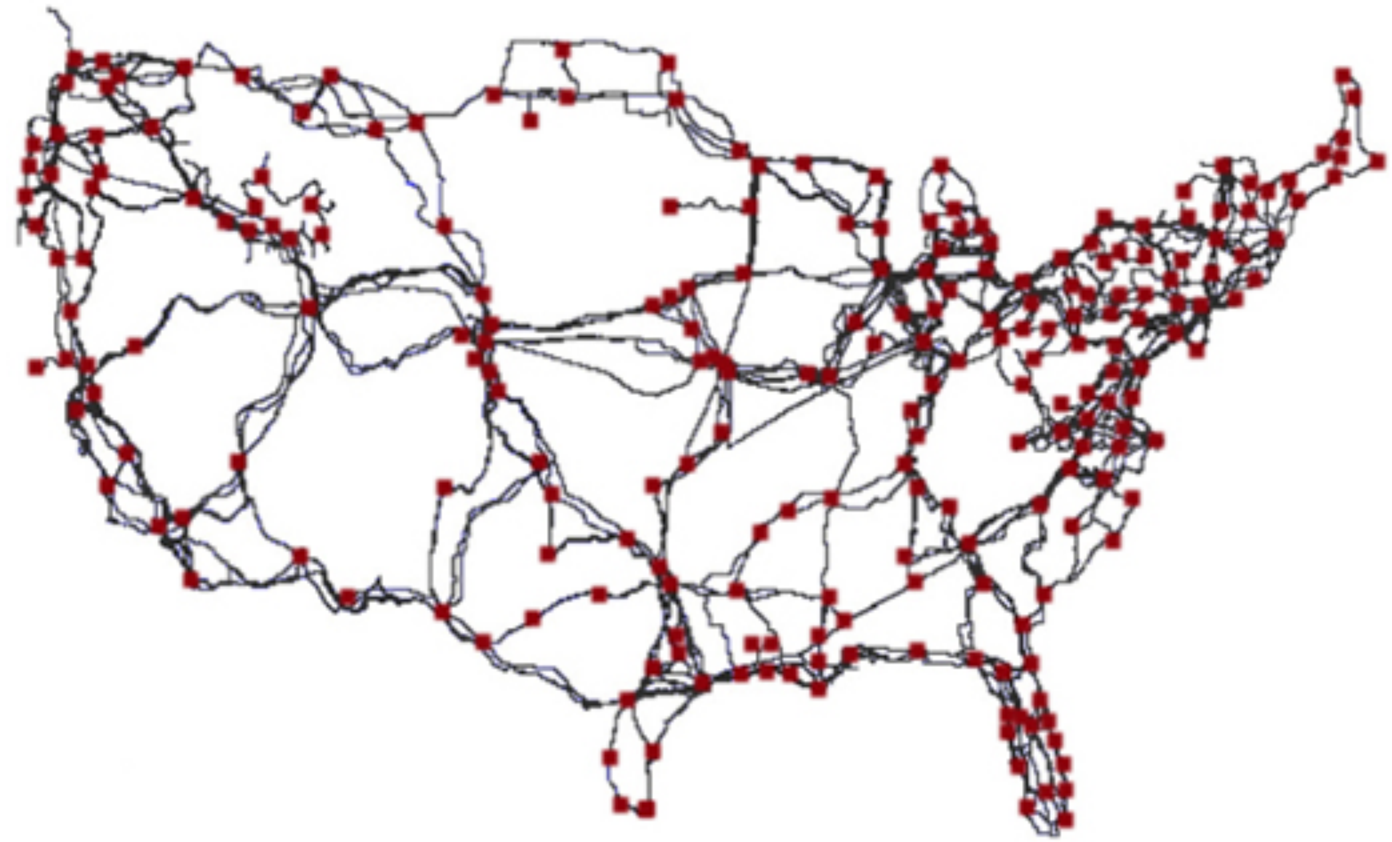
Examples

- Evolution of gene regulatory and metabolic networks
- Self organization, adaptation of food webs
- Social network formation and change, growth of collaboration and citation networks
- Global economic relationships, trade, diplomacy, etc.
- Growth of infrastructure networks (power grids, sanitation, traffic, railways, internet)
- Many of these are potentially adaptive networks



Rail network

Internet fiber cable network

https://en.wikipedia.org/wiki/Rail_transportation_in_the_United_States

https://www.technologyreview.com/s/540721/first-detailed-public-map-of-us-internet-backbone-could-make-it-stronger/

For next time…

- Reading
  - Sayama Chapter 16
  - Think Complexity Chapters 4 & 5